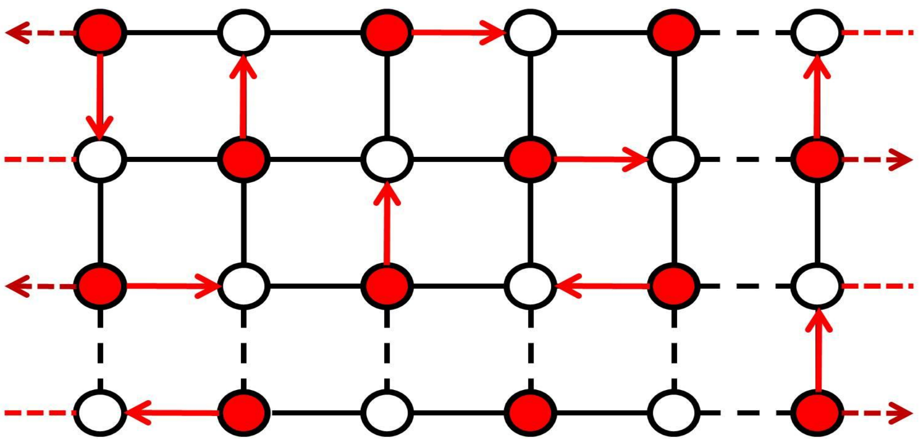
BIPARTITE AND MULTIPARTITE ENTANGLEMENT IN RESONATING VALENCE BOND LADDERS



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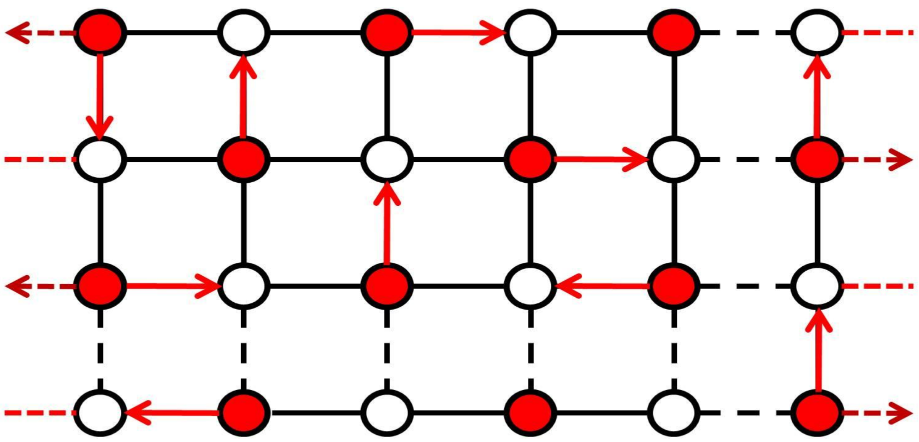
ISCQI, Bhubaneshwar 2011

BIPARTITE AND MULTIPARTITE ENTANGLEMENT IN RESONATING VALENCE BOND LADDERS

USING THE DENSITY MATRIX RECURSION METHOD



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OUTLINE

- INTRODUCTION
- RESONATING VALENCE BOND LADDERS
- DENSITY MATRIX RECURSION METHOD
- CALCULATING ENTANGLEMENT
- ENTANGLEMENT PROPERTIES OF RVB LADDERS
- CONCLUSION

INTRODUCTION

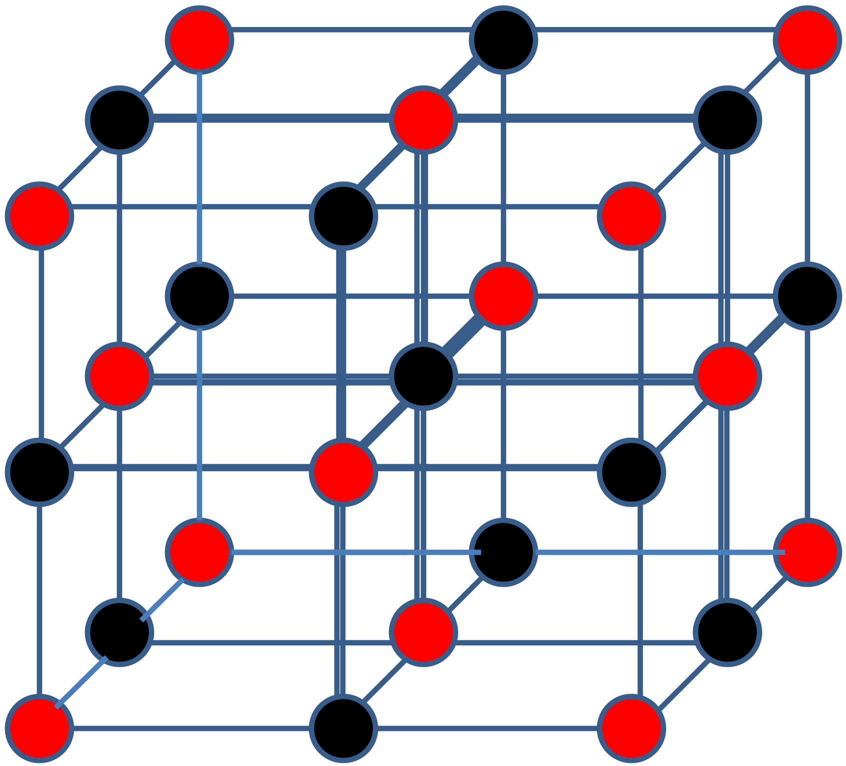
RESONATING VALENCE BOND (RVB) STATES

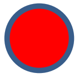
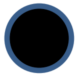
- INTRODUCED BY LINUS PAULING IN 1938
- P. W. ANDERSON, IN 1973, INTRODUCED RVB IN CONDENSED MATTER PHYSICS TO EXPLAIN MOTT INSULATORS
- RVB STATES WERE RELATED TO HIGH TEMPERATURE SUPERCONDUCTIVITY
- POSSIBLE USE IN TOPOLOGICAL QUANTUM COMPUTATION[1]

[1]A. Y. Kitaev, *"Fault-tolerant quantum computation by anyons"*. Ann. Phys., Lpz. **303**, 2 (2003)

INTRODUCTION

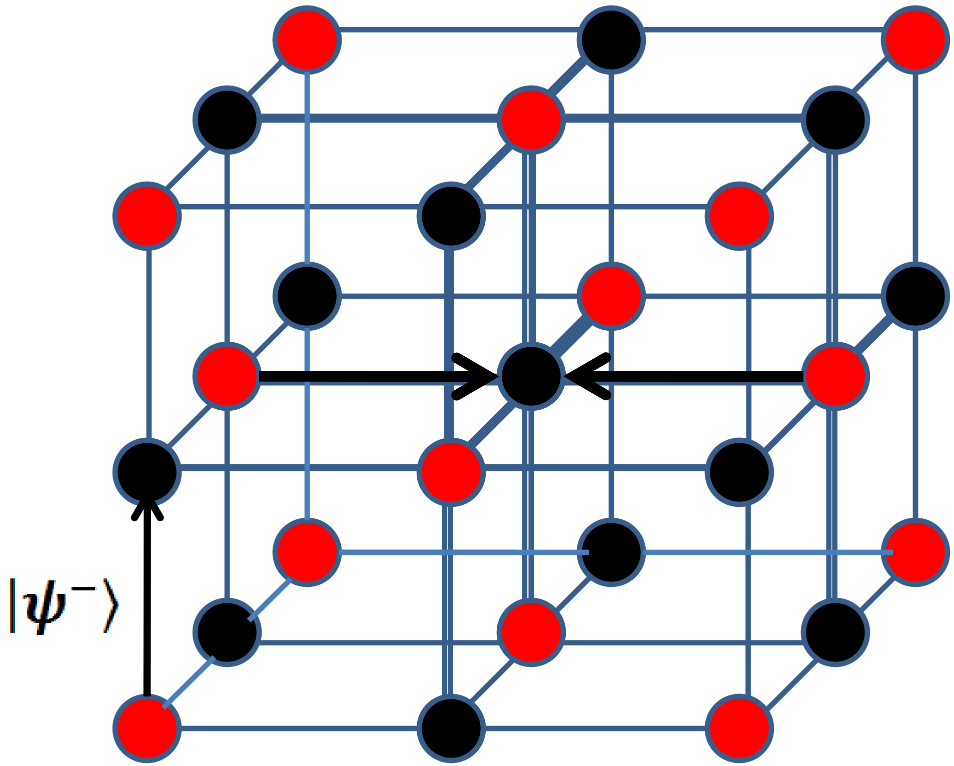
RVB STATES USING BIPARTITE LATTICE

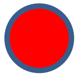
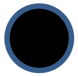


RVB BIPARTITE LATTICE **SUBLATTICE : A**  **B** 

INTRODUCTION

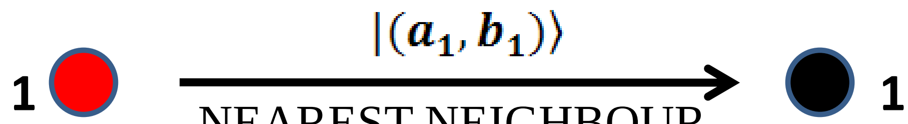
RVB STATES USING BIPARTITE LATTICE



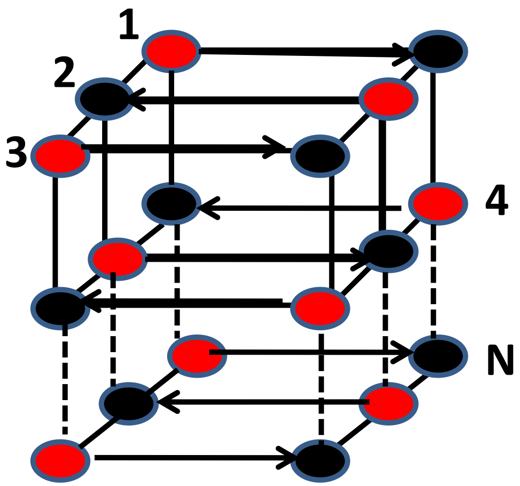
RVB BIPARTITE LATTICE **SUBLATTICE : A**  **B** 

INTRODUCTION

THE RVB STATE



NEAREST NEIGHBOUR
SINGLET PAIR

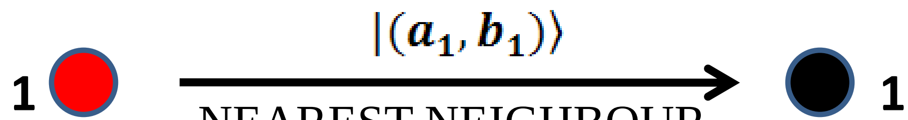


$$\left[\prod_{(i,j)'} |(a_i, b_j)\rangle\right]_k = |(a_1, b_1)\rangle|(a_2, b_2)\rangle \dots |(a_N, b_N)\rangle$$

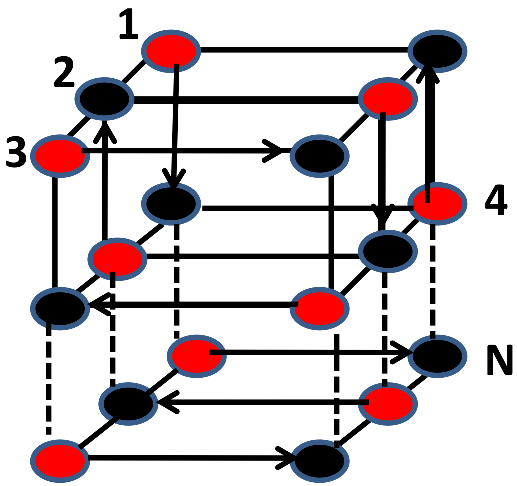
ONE POSSIBLE COMBINATION

INTRODUCTION

THE RVB STATE



NEAREST NEIGHBOUR
SINGLET PAIR



$$\left[\prod_{(i,j)'} |(a_i, b_j)\rangle\right]_k = |(a_1, b_4)\rangle|(a_2, b_5)\rangle \dots |(a_N, b_N)\rangle$$

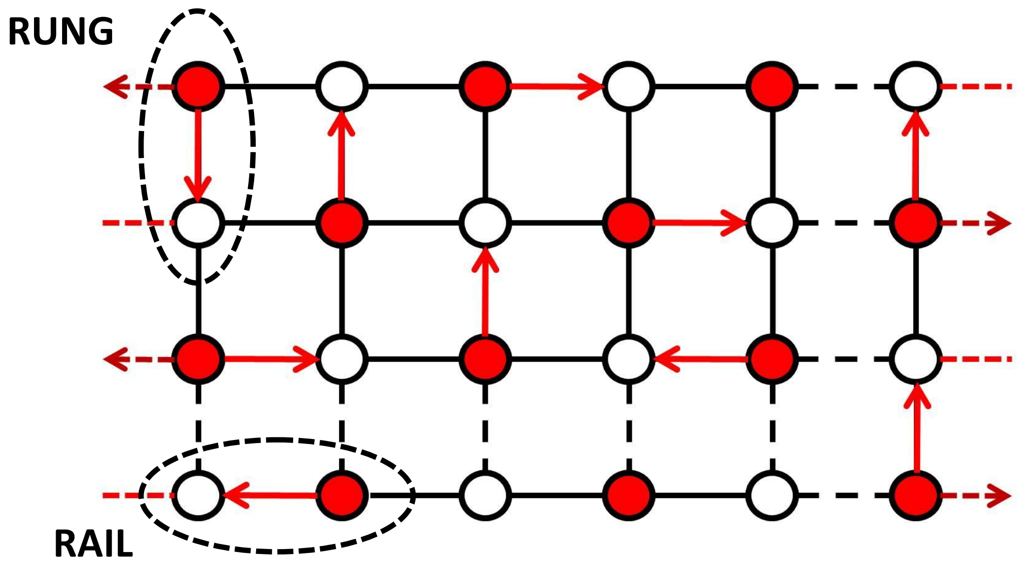
ANOTHER POSSIBLE COMBINATION

$$\sum_k \left[\prod_{(i,j)'} |(a_i, b_j)\rangle\right]_k$$

SUPERPOSITION OF ALL
SUCH COMBINATIONS

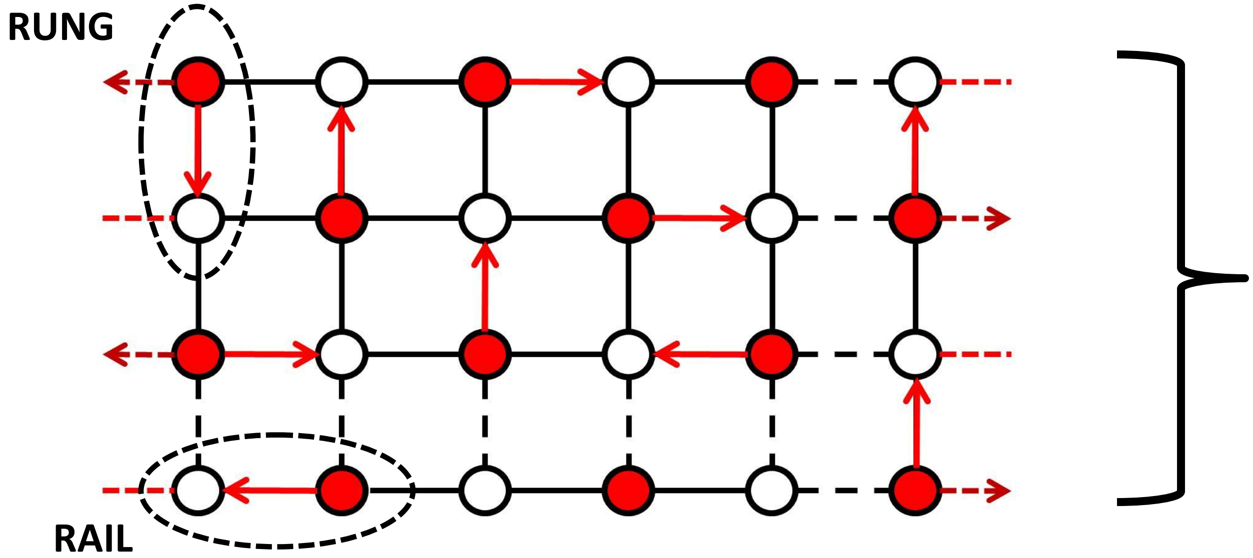
RESONATING VALENCE BOND LADDERS

INTERESTS



RESONATING VALENCE BOND LADDERS

INTERESTS

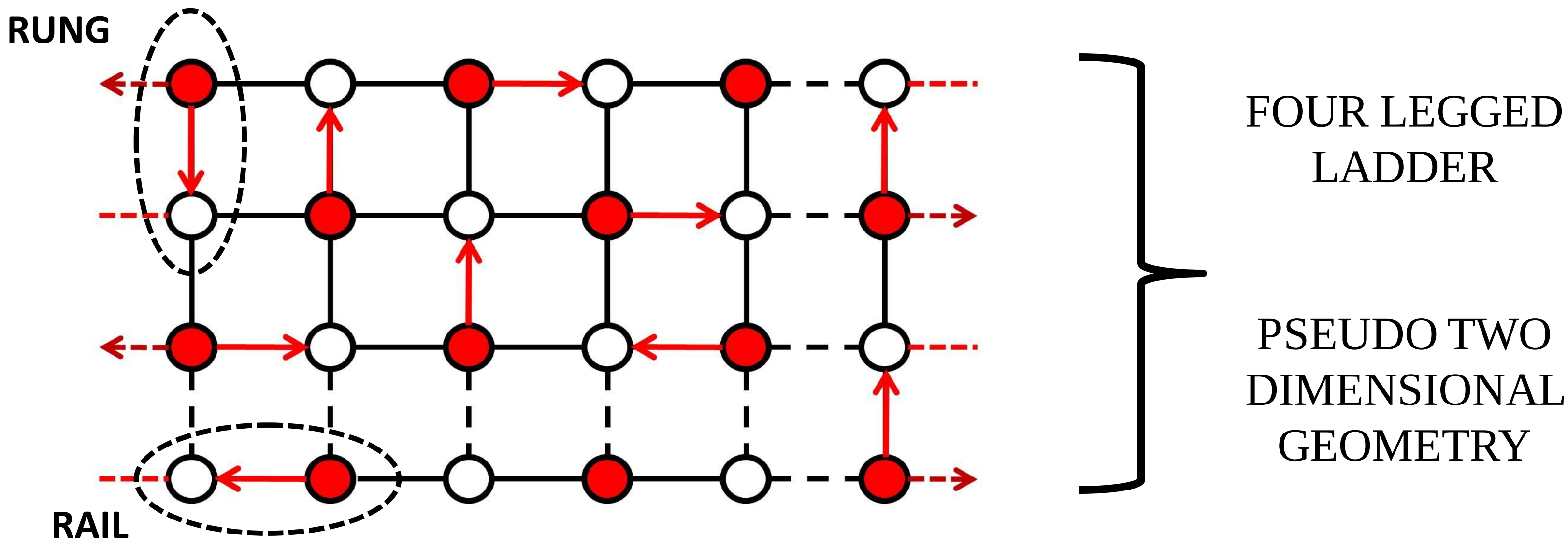


FOUR LEGGED LADDER

PSEUDO TWO DIMENSIONAL GEOMETRY

RESONATING VALENCE BOND LADDERS

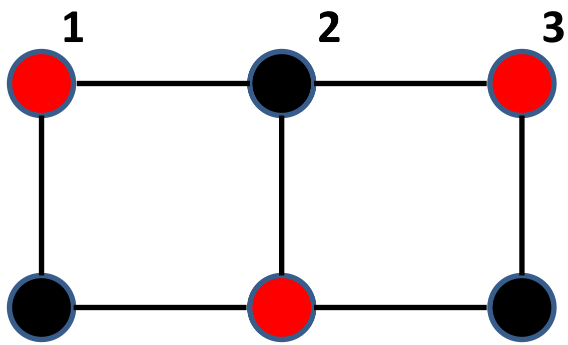
INTERESTS



- POSSIBLE GROUND STATES OF THE ANTIFERROMAGNETIC HEISENBERG MODEL
- EVEN LEGGED LADDERS RELATED TO HIGH – T_C SUPERCONDUCTIVITY
- **BIPARTITE AND MULTIPARTITE ENTANGLEMENT PROPERTIES IN RVB LADDER SYSTEMS HAVE INTERESTING PROPERTIES DEPENDENT ON GEOMETRY**

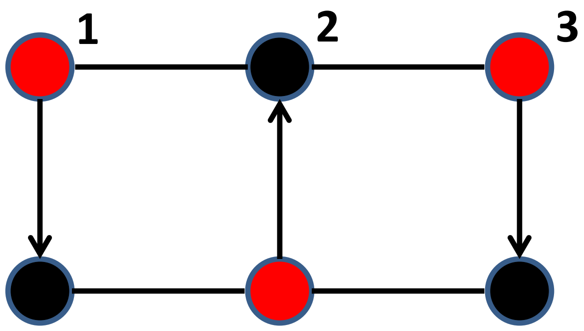
RESONATING VALENCE BOND LADDERS

OBTAINING THE RVB STATE



2 X 3 LADDER

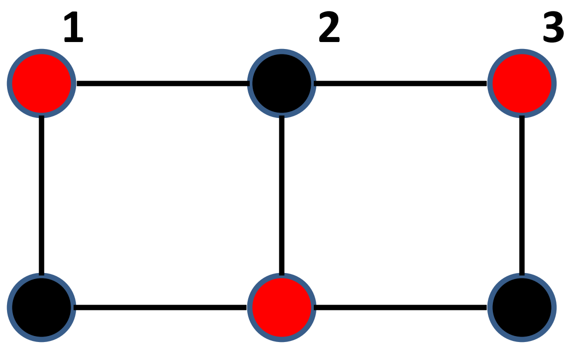
(NON PERIODIC BOUNDARY CONDITION)



$$|(a_1, b_1)\rangle|(a_2, b_2)\rangle|(a_3, b_3)\rangle$$

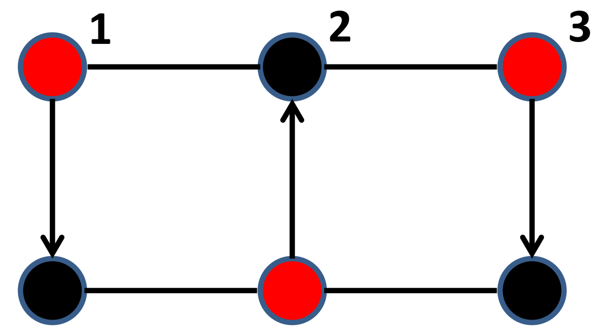
RESONATING VALENCE BOND LADDERS

OBTAINING THE RVB STATE

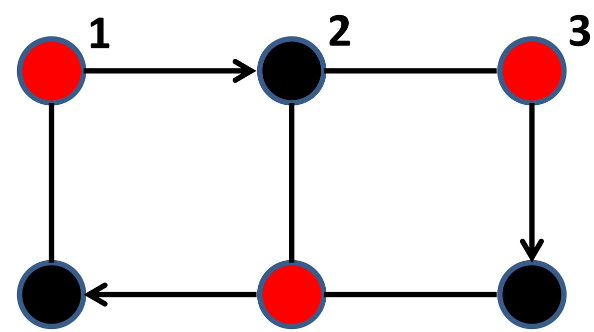


2 X 3 LADDER

(NON PERIODIC BOUNDARY CONDITION)



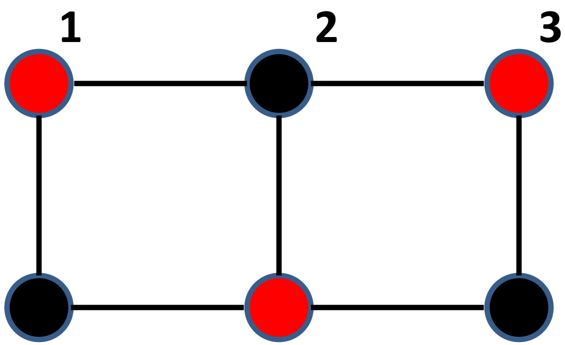
$$|(a_1, b_1)\rangle|(a_2, b_2)\rangle|(a_3, b_3)\rangle$$



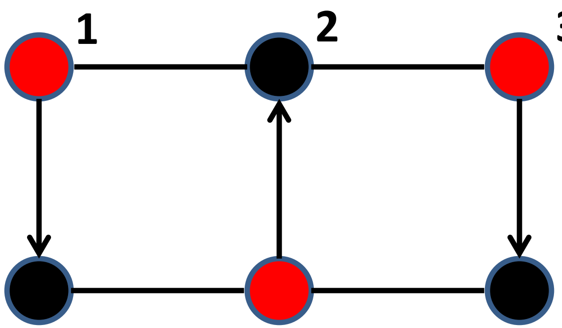
$$|(a_1, b_2)\rangle|(a_2, b_1)\rangle|(a_3, b_3)\rangle$$

RESONATING VALENCE BOND LADDERS

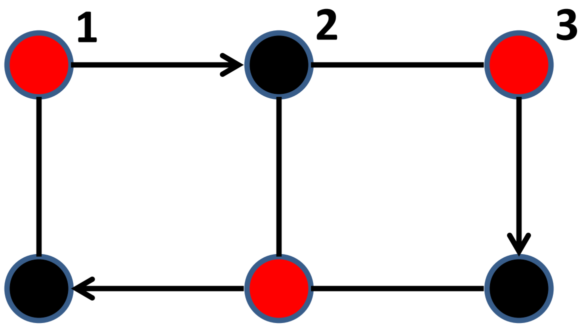
OBTAINING THE RVB STATE



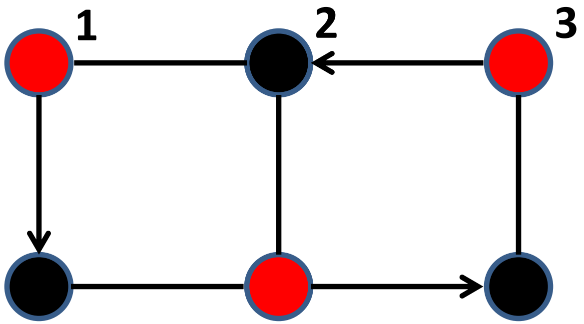
2 X 3 LADDER
(NON PERIODIC BOUNDARY CONDITION)



$$|(a_1, b_1)\rangle|(a_2, b_2)\rangle|(a_3, b_3)\rangle$$



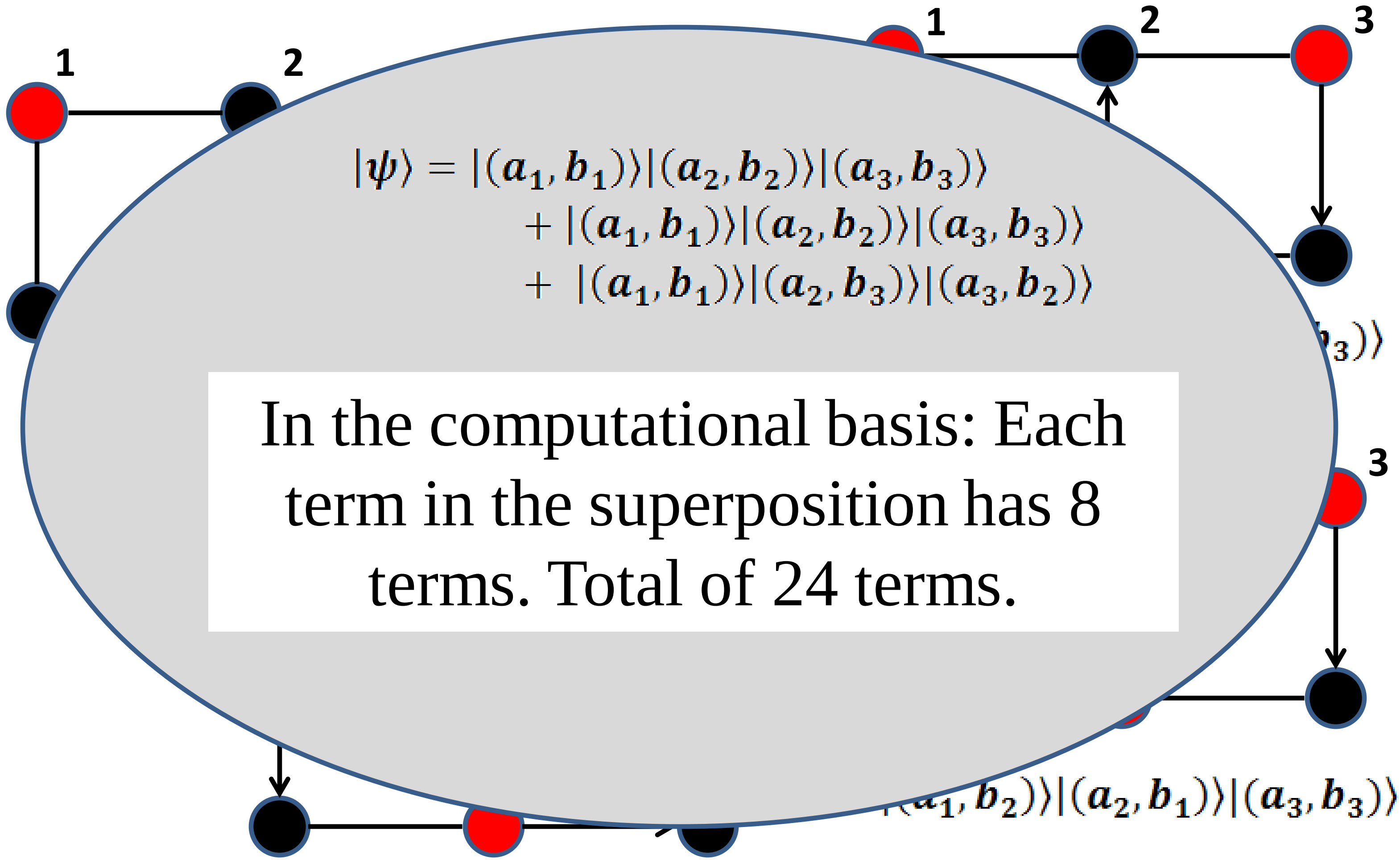
$$|(a_1, b_2)\rangle|(a_2, b_1)\rangle|(a_3, b_3)\rangle$$



$$|(a_1, b_1)\rangle|(a_2, b_3)\rangle|(a_3, b_2)\rangle$$

RESONATING VALENCE BOND LADDERS

OBTAINING THE RVB STATE

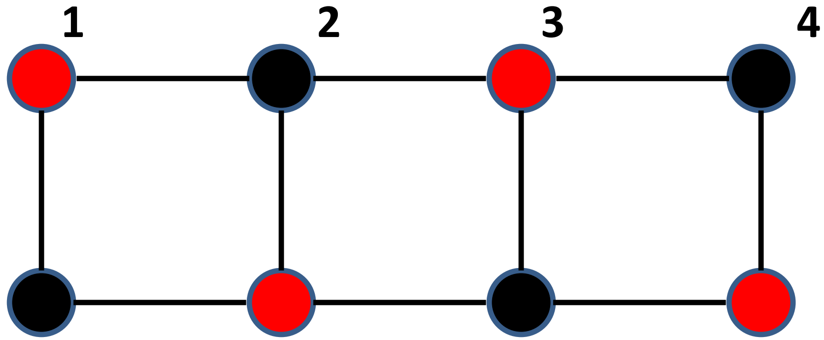


$$|\psi\rangle = |(a_1,b_1)\rangle|(a_2,b_2)\rangle|(a_3,b_3)\rangle + |(a_1,b_1)\rangle|(a_2,b_2)\rangle|(a_3,b_3)\rangle + |(a_1,b_1)\rangle|(a_2,b_3)\rangle|(a_3,b_2)\rangle$$

In the computational basis: Each term in the superposition has 8 terms. Total of 24 terms.

RESONATING VALENCE BOND LADDERS

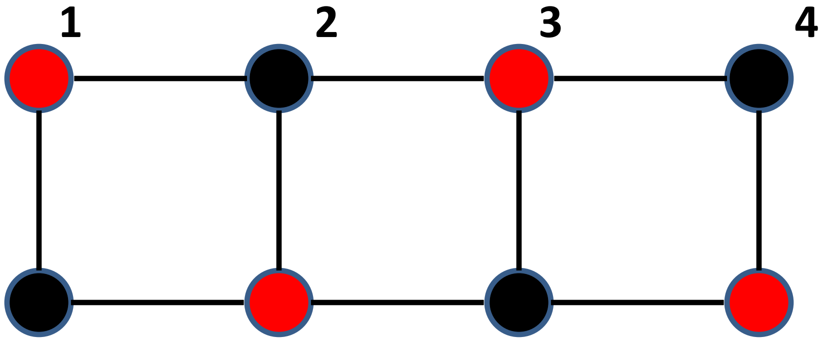
OBTAINING THE RVB STATE



5 terms in the superposition, with 16 terms in each. TOTAL=80 terms.

RESONATING VALENCE BOND LADDERS

OBTAINING THE RVB STATE



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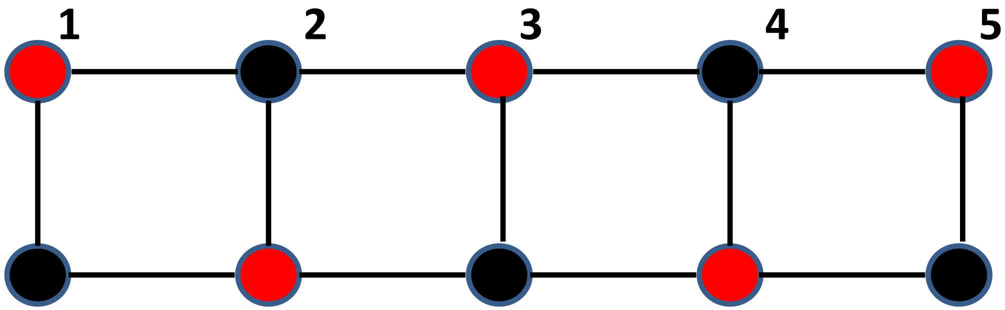
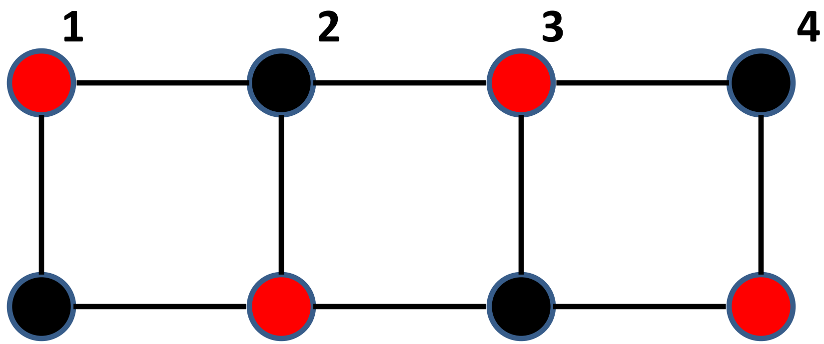


8 terms in the superposition, with 32 terms in each. TOTAL = 256 terms.

RESONATING VALENCE BOND LADDERS

OBTAINING THE RVB STATE



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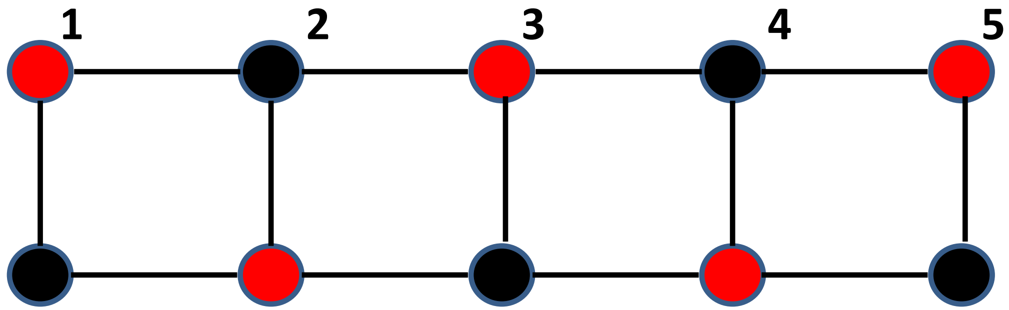

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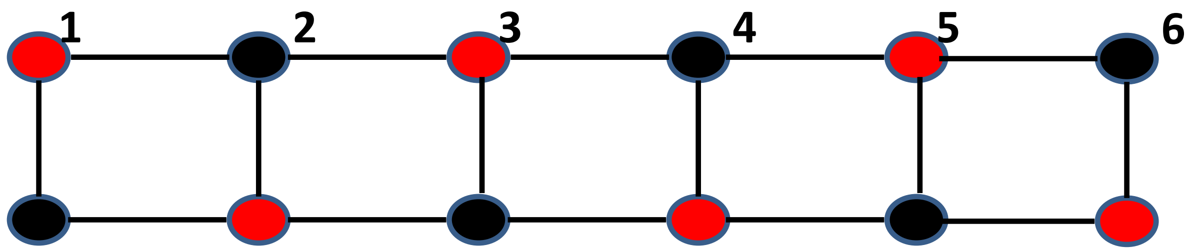
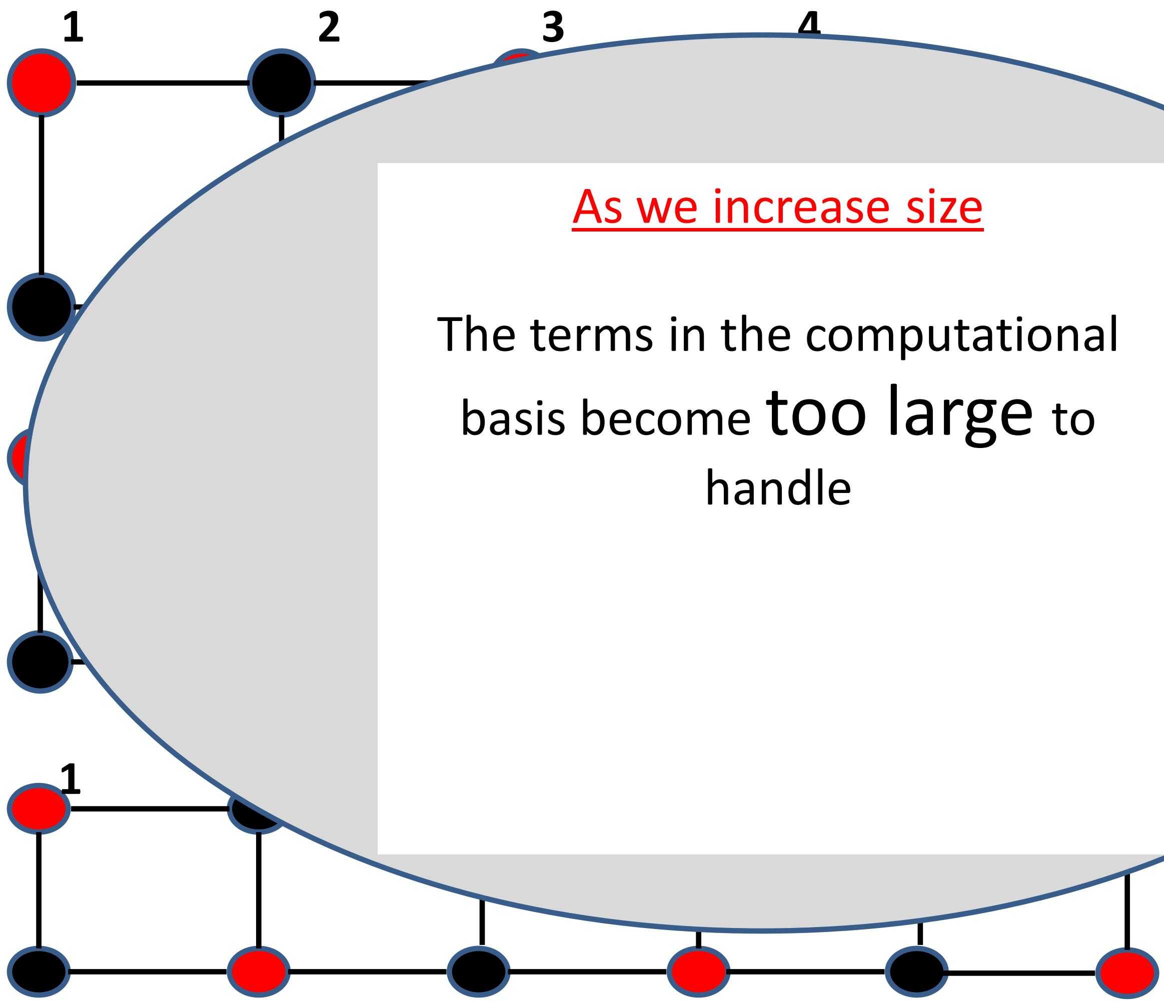


11 terms in the superposition, with 64 terms in each. TOTAL = 704 terms.

RESONATING VALENCE BOND LADDERS

OBTAINING THE RVB STATE



the superposition,
s in each.
ns.

As we increase size

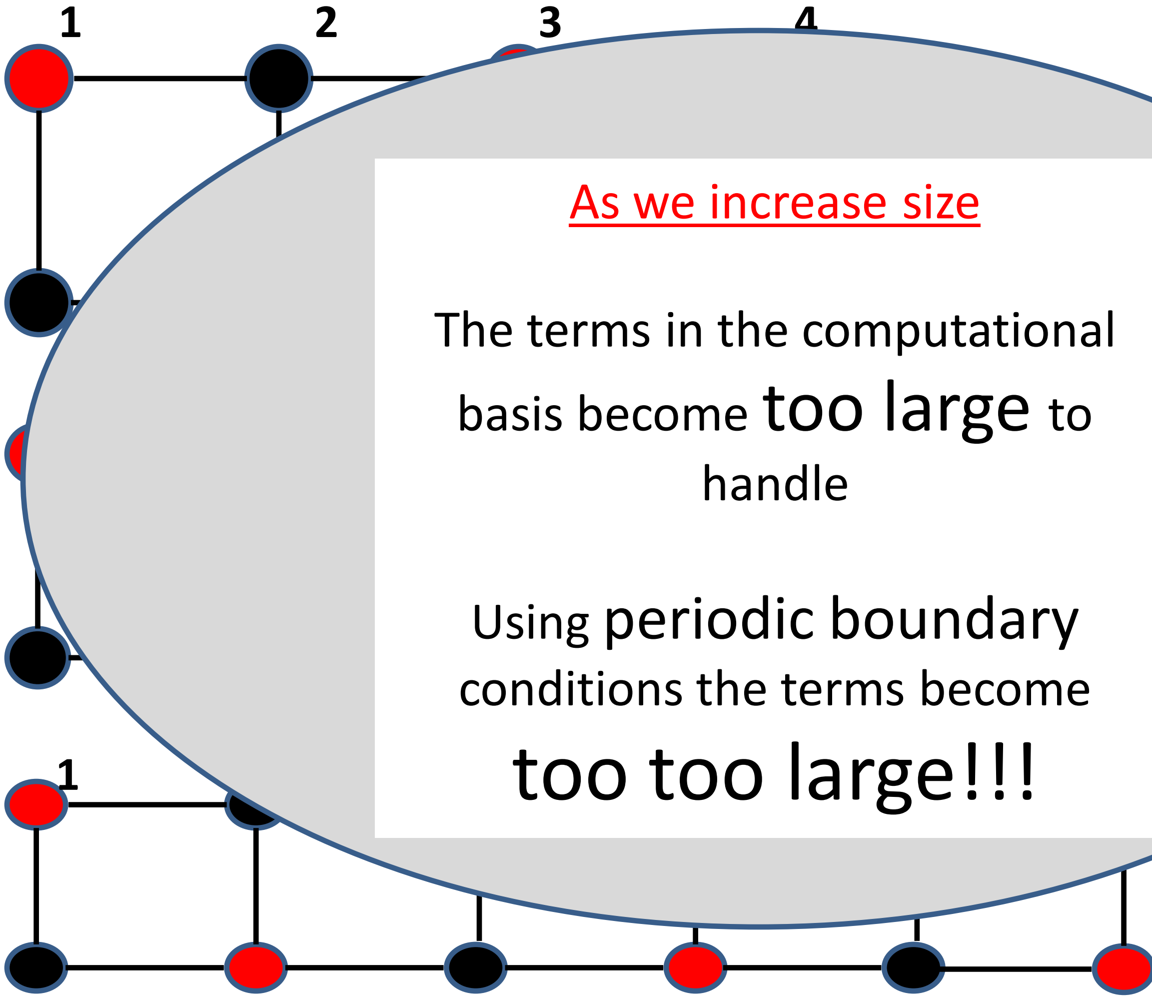
The terms in the computational basis become **too large** to handle

32
AL =

terms in the superposition, with 64 terms in each. TOTAL = 704 terms.

RESONATING VALENCE BOND LADDERS

OBTAINING THE RVB STATE



the superposition,
s in each.
ns.

32
AL =

terms in the superposition, with 64 terms in each. TOTAL = 704 terms.

As we increase size

The terms in the computational basis become too large to handle

Using periodic boundary conditions the terms become too too large!!!

DENSITY MATRIX RECURSION METHOD[1]

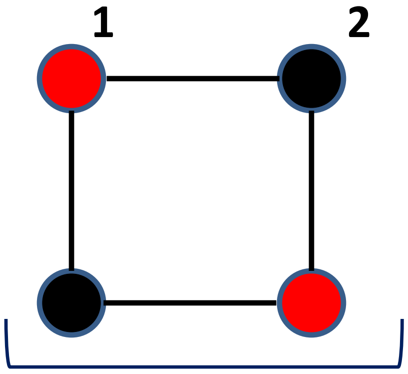
USING DMRM TO CALCULATE RVB LADDER STATES

- RECURSIVE METHOD TO CALCULATE THE RVB STATE FOR LARGE EVEN RUNG LADDER FROM SMALLER UNITS
- CALCULATIONS CAN BE EXTENDED TO LARGE NUMBER OF RUNGS
- ENABLES ANALYTICAL EXPRESSIONS TO COMPUTE BIPARTITE AND MULTIPARTITE ENTANGLEMENT FOR LARGE LADDERS.
- CAN BE USED TO CALCULATE OTHER FORMS OF CORRELATIONS

[1] H. S. Dhar, A. Sen (De) and U. Sen, *Density Matrix Recursion Method: Genuine Multisite Entanglement Distinguishes Odd from Even Quantum Heisenberg Ladders,* arXiv:1110.3646v1 [quant-ph] (2011).

DENSITY MATRIX RECURSION METHOD

USING DMRM TO CALCULATE RVB STATES

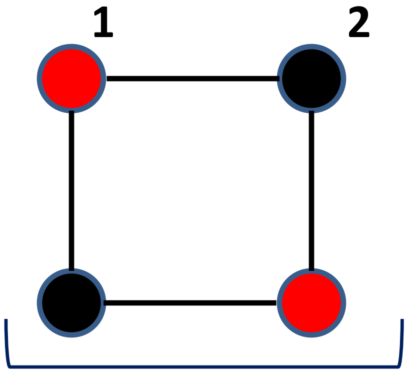


$|2\rangle_{12}$

BASIC BLOCK

DENSITY MATRIX RECURSION METHOD

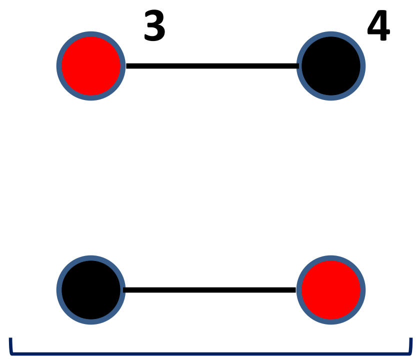
USING DMRM TO CALCULATE RVB STATES



$|2\rangle_{12}$

BASIC BLOCK

+



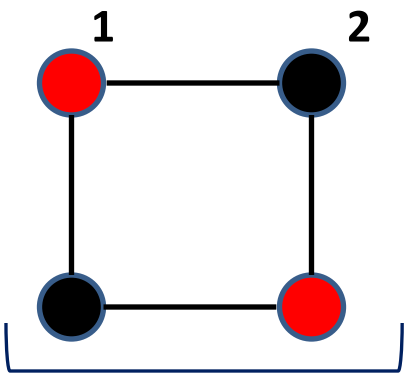
$|2`\rangle_{34}$

ADDITIONAL BLOCK

OBTAINING A NON PERIODIC 4 RUNG LADDER FROM 2 AND 3 RUNG LADDER

DENSITY MATRIX RECURSION METHOD

USING DMRM TO CALCULATE RVB STATES



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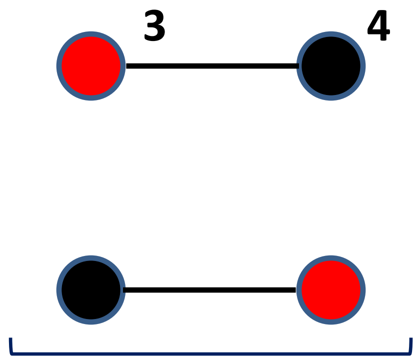


$|2\rangle_{12}$

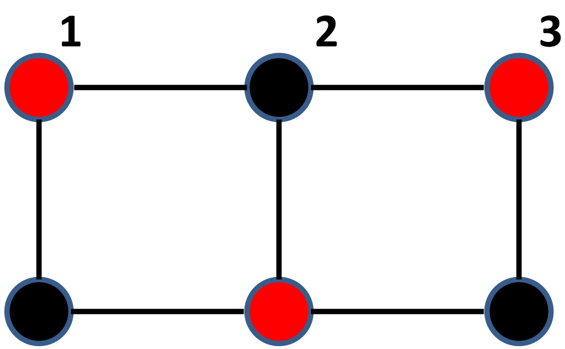
BASIC BLOCK

$|2`\rangle_{34}$

ADDITIONAL BLOCK

OBTAINING A NON PERIODIC 4 RUNG LADDER FROM 2 AND 3 RUNG LADDER

+



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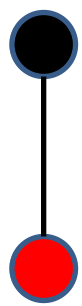
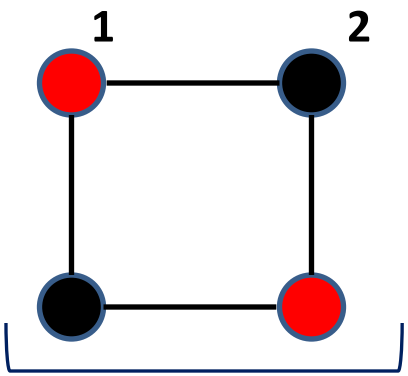


DENSITY MATRIX RECURSION METHOD

USING DMRM TO CALCULATE RVB STATES



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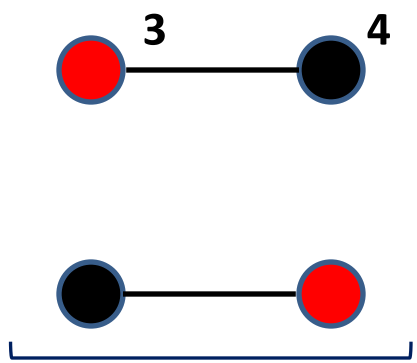


$|2\rangle_{12}$

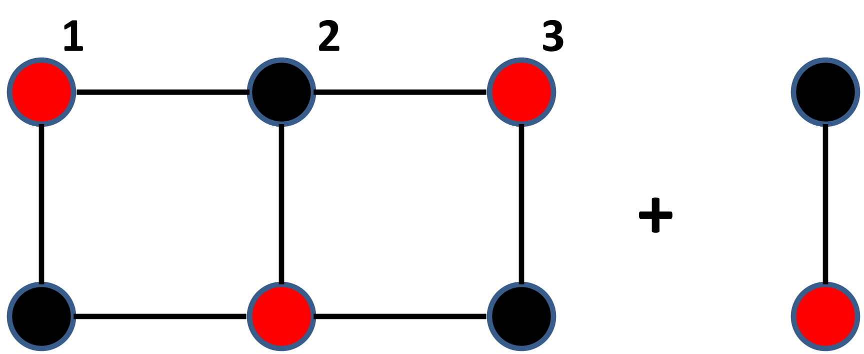
BASIC BLOCK

$|2`\rangle_{34}$

ADDITIONAL BLOCK

OBTAINING A NON PERIODIC 4 RUNG LADDER FROM 2 AND 3 RUNG LADDER

+

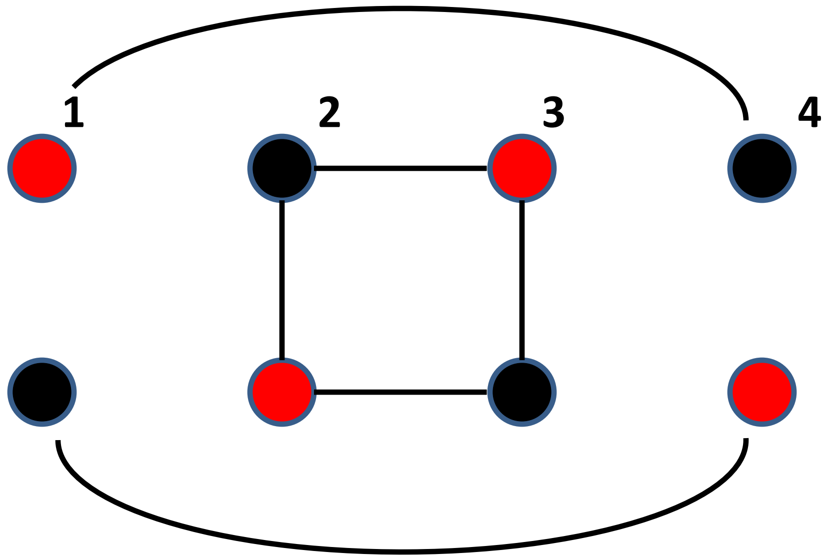


$$|4\rangle_{1..4} = |3\rangle_{1..3}|1\rangle_4 + |2\rangle_{12}|2'\rangle_{34}$$

DENSITY MATRIX RECURSION METHOD

USING DMRM TO CALCULATE RVB STATES

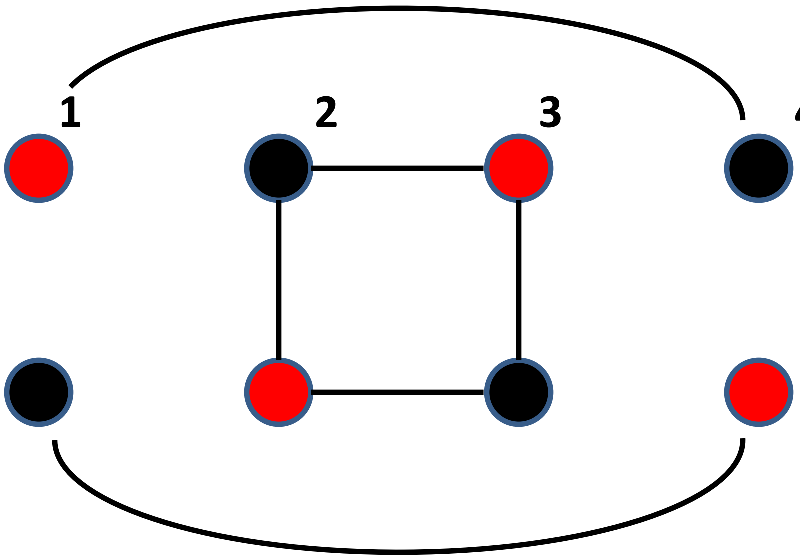
USING PERIODIC BOUNDARY CONDITIONS:



DENSITY MATRIX RECURSION METHOD

USING DMRM TO CALCULATE RVB STATES

USING PERIODIC BOUNDARY CONDITIONS:

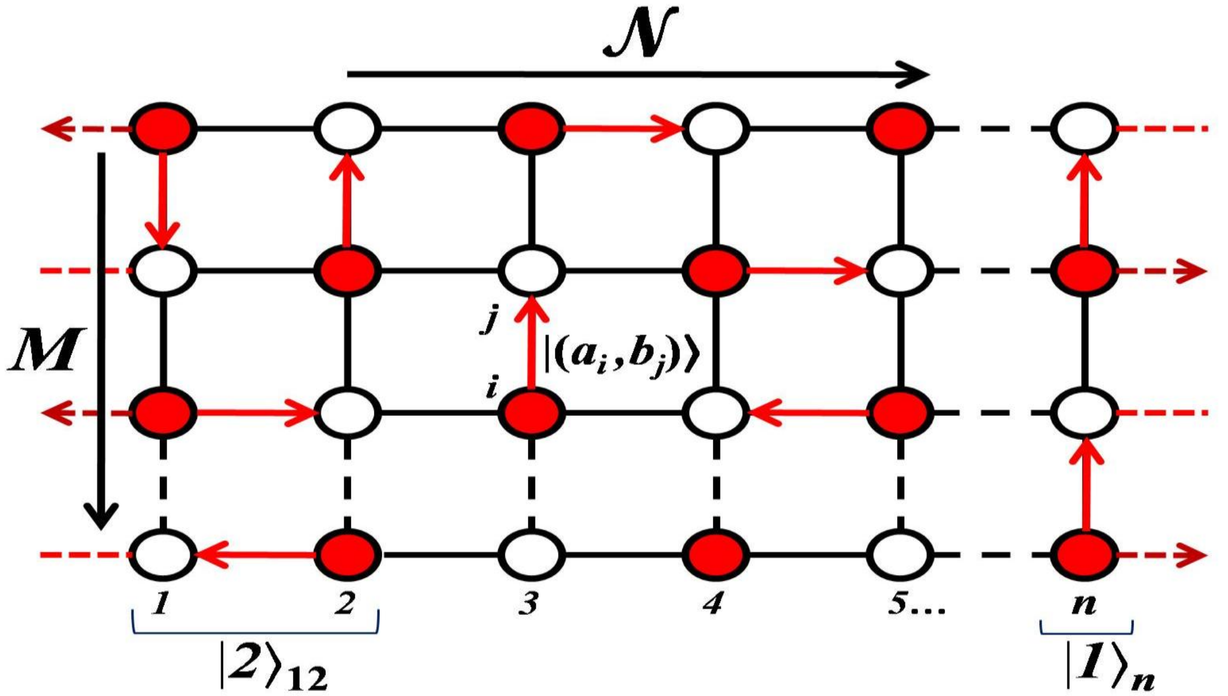


$$|4\rangle_{1..4}^{p} = |3\rangle_{1..3}|1\rangle_4 + |2\rangle_{12}|2'\rangle_{34} + |2\rangle_{23}|2'\rangle_{14}$$

DENSITY MATRIX RECURSION METHOD

USING DMRM TO CALCULATE RVB STATES

GENERALIZING FOR MULTI-LEG PERIODIC LADDER :



DENSITY MATRIX RECURSION METHOD

USING DMRM TO CALCULATE RVB STATES

GENERALIZING FOR MULTI-LEG PERIODIC LADDER :

EVEN LEG

$$|N+2\rangle_{1..N+2} = |N\rangle_N |2'\rangle_{N+1,N+2} + |N+1\rangle_{1..N+1} |1\rangle_{N+2}$$

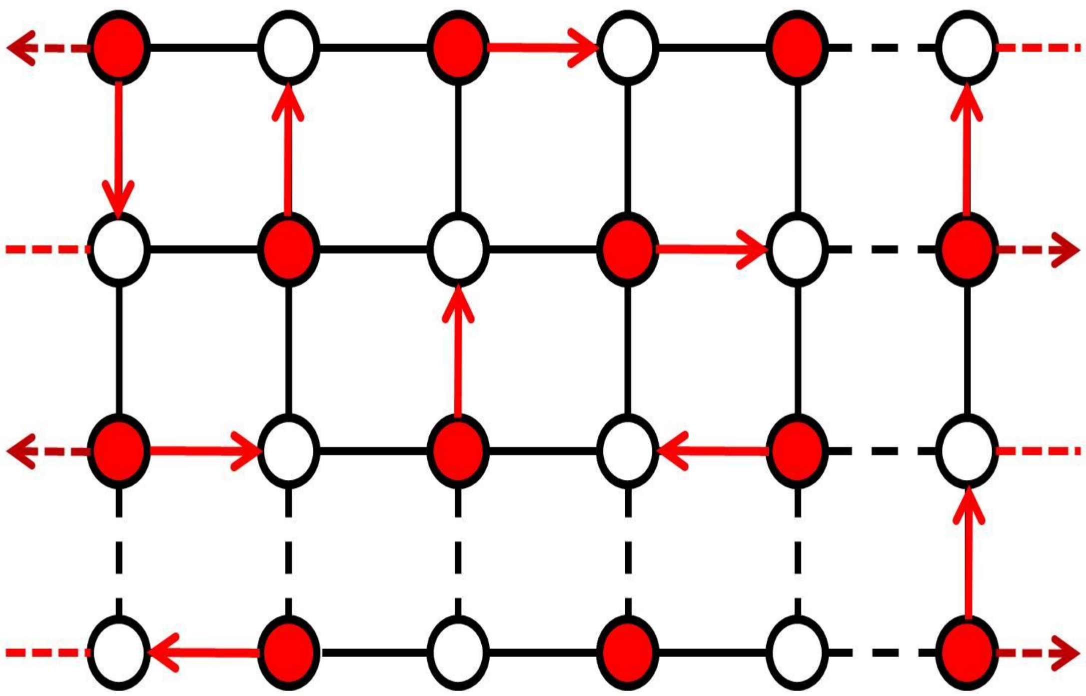
$$|N+2\rangle^p_{1..N+2} = |N+2\rangle_{1..N+2} + |N\rangle_{2..N+1} |2\rangle_{N+2,1}$$

ODD LEG

$$|N+2\rangle^p_{1..N+2} = |N\rangle_{1..N} |2\rangle_{N+1,N+2} + |N\rangle_{2..N+1} |2\rangle_{N+2,1}$$

CALCULATING ENTANGLEMENT

CONVENTIONAL APPROACH

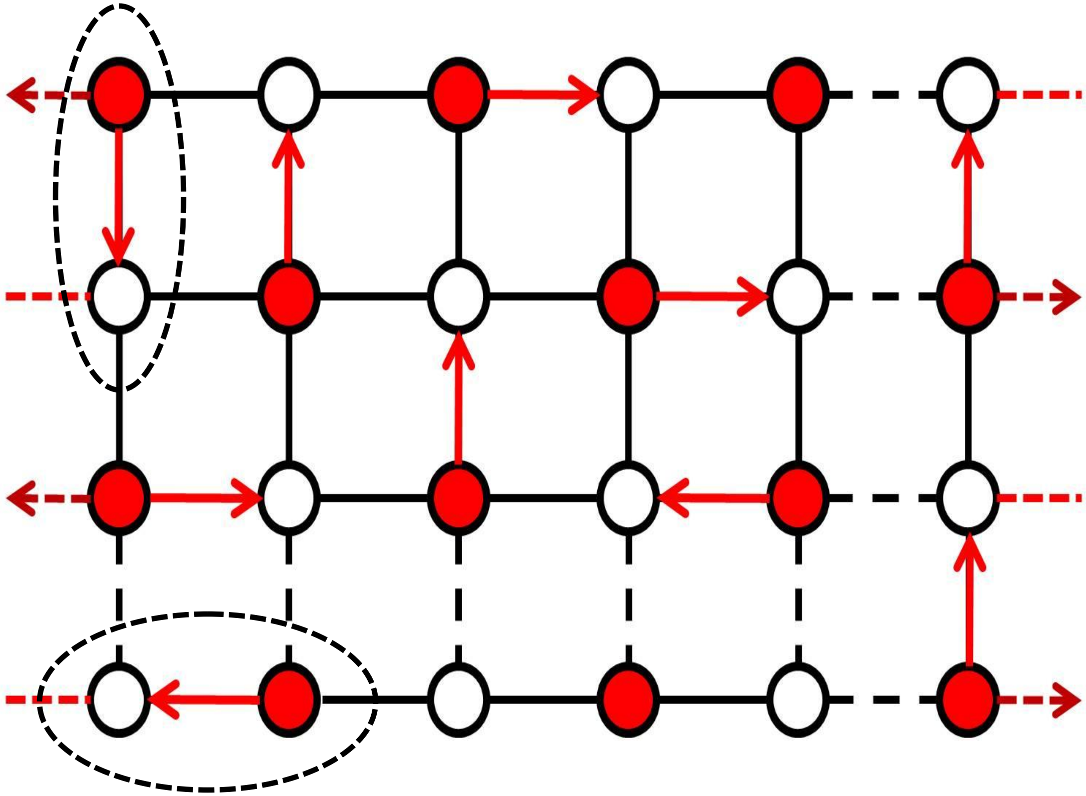


ROTATIONALLY INVARIANT

PERIODIC BOUNDARY CONDITIONS INTRODUCE A SYMMETRY

CALCULATING ENTANGLEMENT

CONVENTIONAL APPROACH



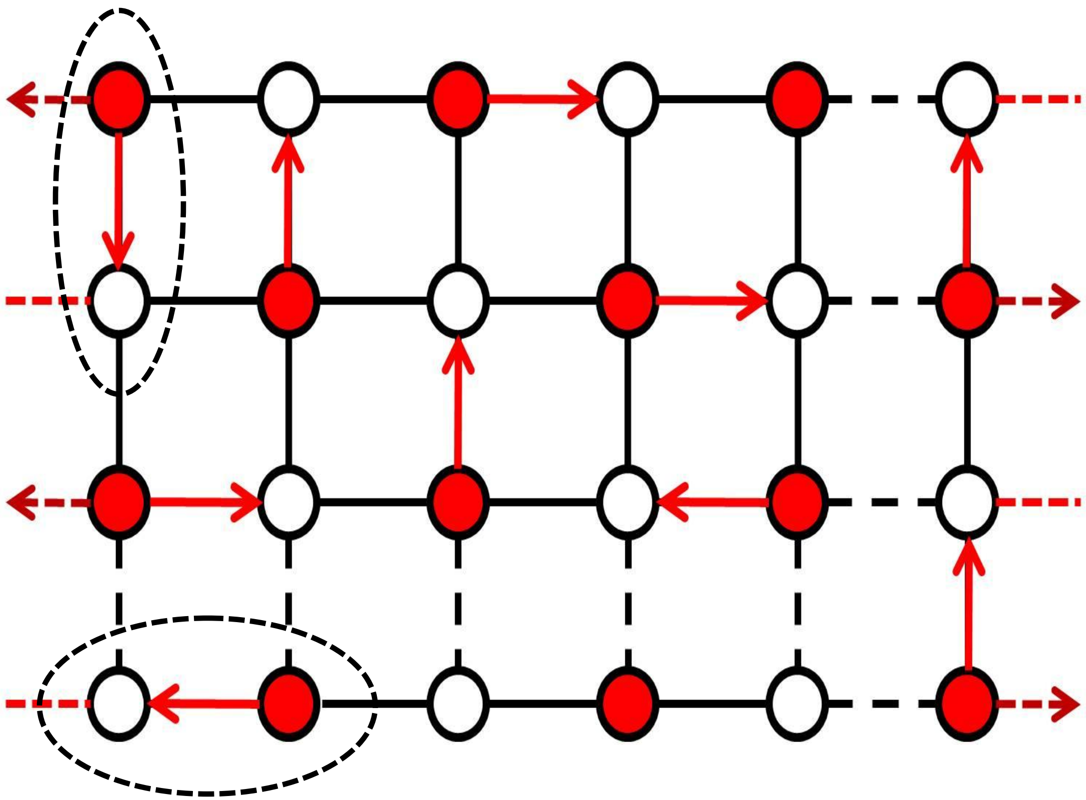
FOR BIPARTITE ENTANGLEMENT:

TRACE OUT ALL BUT ANY TWO SITES FROM THE DENSITY MATRIX OF THE RVB LADDER

OBTAIN THE TWO-SITE REDUCED DENSITY MATRIX

CALCULATING ENTANGLEMENT

CONVENTIONAL APPROACH



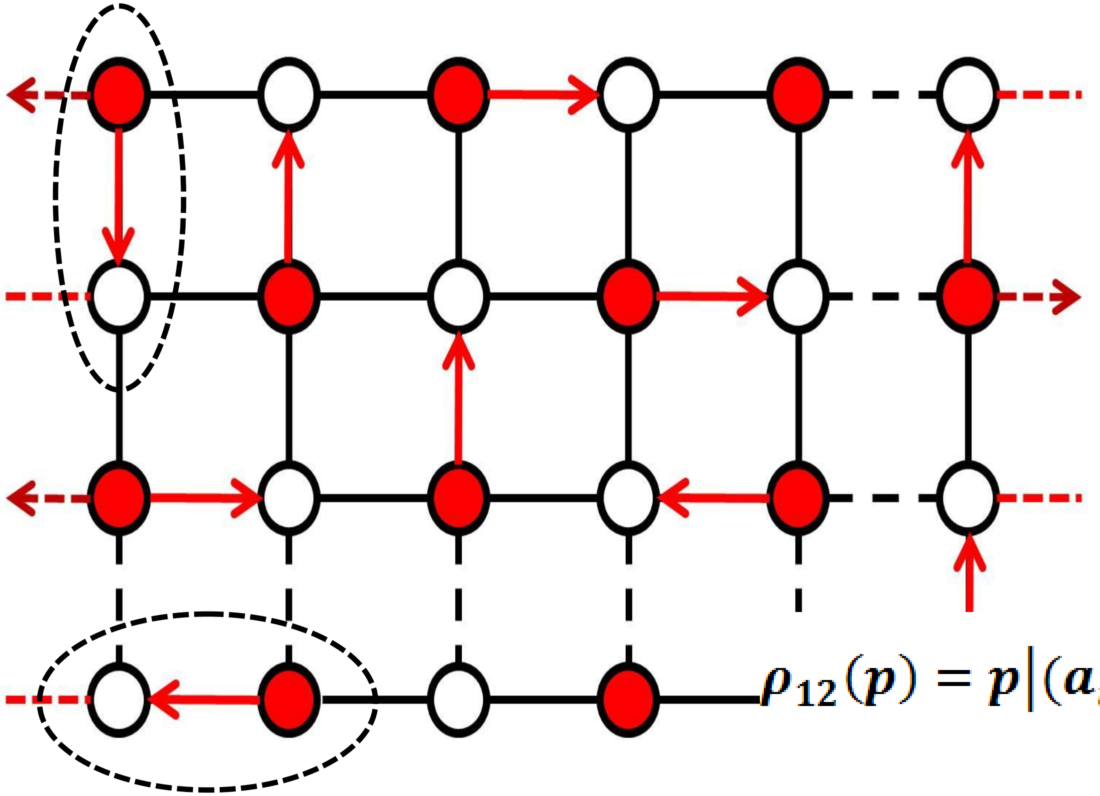
FOR BIPARTITE ENTANGLEMENT:

TWO SITE DENSITY MATRIX IS ROTATIONALLY INVARIANT

$$\rho_{12} = \mathbf{Tr}_{\overline{12}} |N\rangle\langle N|$$

CALCULATING ENTANGLEMENT

CONVENTIONAL APPROACH



FOR BIPARTITE ENTANGLEMENT:

TWO SITE DENSITY MATRIX IS ROTATIONALLY INVARIANT

$$\boldsymbol{\rho_{12} = \mathrm{Tr}_{\overline{12}} |N\rangle\langle N|}$$

HENCE WERNER STATE

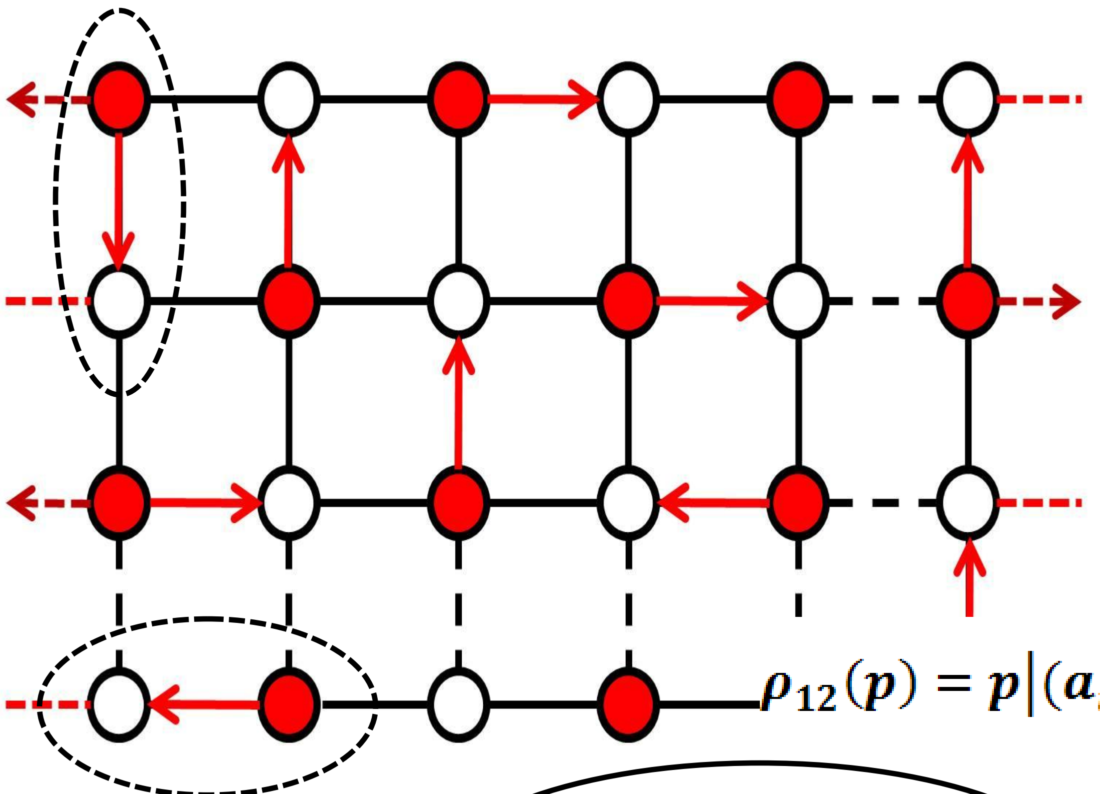
$$\rho_{12}(p) = p\left|(a_i, b_j)\rangle\langle(a_i, b_j)\right| + \frac{1-p}{4} I_4$$

$$-{}^{1}/_{3} \leq p \leq 1$$

WERNER PARAMETER CAN BE USED TO CALCULATE ENTANGLEMENT

CALCULATING ENTANGLEMENT

CONVENTIONAL APPROACH



FOR BIPARTITE ENTANGLEMENT:

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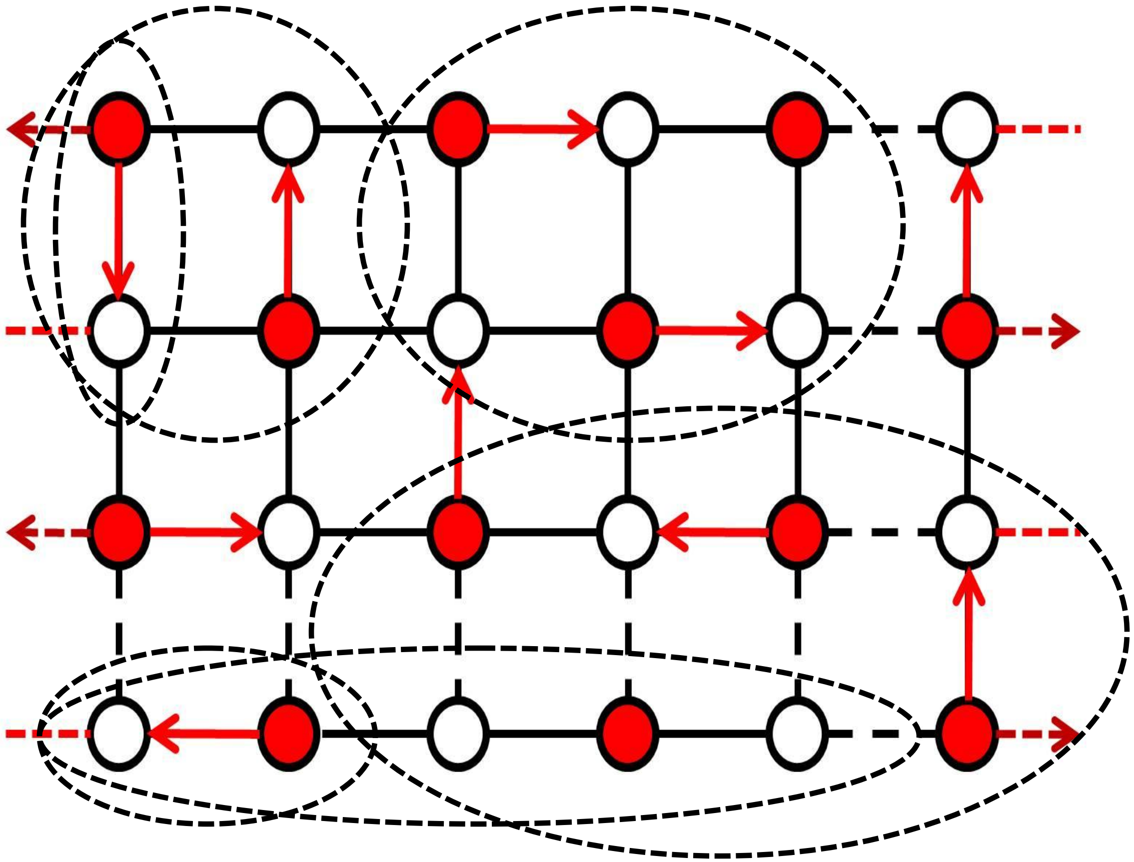
$$-{}^{1}/_{3} \leq p \leq 1$$

$p > {}^{1}/_{3}$ **ENTANGLED**

WERNER PARAMETER CAN BE USED TO CALCULATE ENTANGLEMENT

CALCULATING ENTANGLEMENT

CONVENTIONAL APPROACH



FOR MULTIPARTITE ENTANGLEMENT:

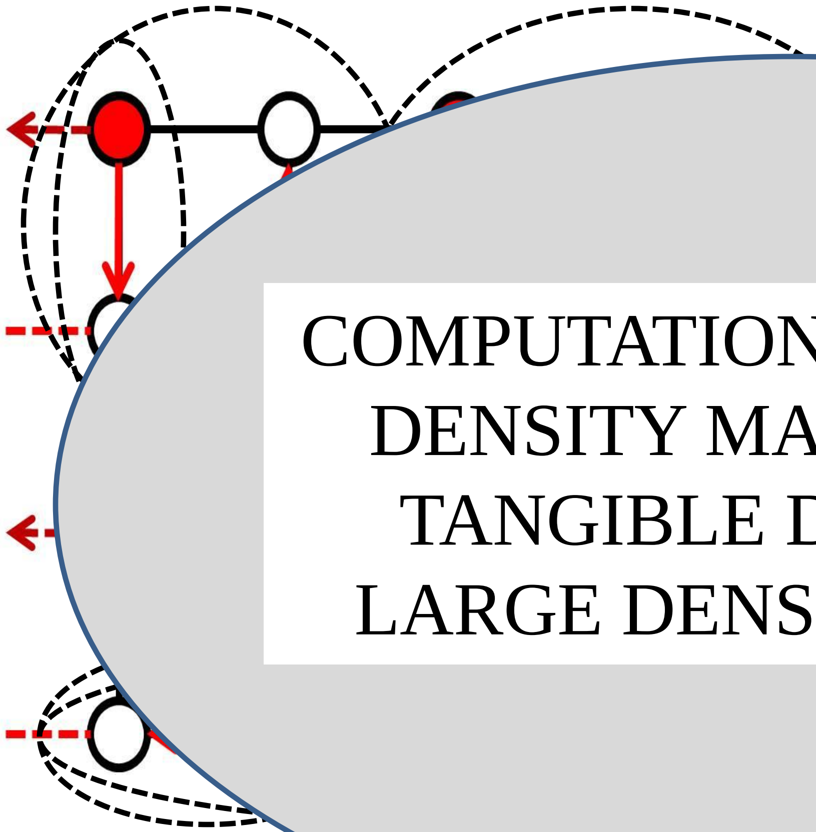
FIND THE 2-SITE, 3-SITE, 4- SITE ….N/2 SITE REDUCED DENSITY MATRIX.

APPLY THE GENERALIZED GEOMETRIC MEASURE (GGM)

$$\varepsilon(|\phi_N\rangle) = 1 - \max\{\lambda^2_{A:B} | A \cup B = \{1, 2, \ldots, N\}, A \cap B = \emptyset\}$$

CALCULATING ENTANGLEMENT

CONVENTIONAL APPROACH



FOR MULTIPARTITE
ANGLEMENT:

SITE, 3-
..N/2

IX.

D

E (GGM)

COMPUTATION OF REDUCED DENSITY MATRIX IS NOT TANGIBLE DUE TO THE LARGE DENSITY MATRIX.

$$\varepsilon(|\phi_N\rangle) = 1 - \max\{\lambda^2_{A:B} | A \cup B = \{1, 2, \ldots, N\}, A \cap B = \emptyset\}$$

CALCULATING ENTANGLEMENT

USING DENSITY MATRIX RECURSION METHOD

H. S. Dhar, A. Sen (De) and U. Sen, *Density Matrix Recursion Method: Genuine Multisite Entanglement Distinguishes Odd from Even Quantum Heisenberg Ladders*, arXiv:1110.3646v1 [quant-ph] (2011).

CALCULATING ENTANGLEMENT

USING DENSITY MATRIX RECURSION METHOD

RECALL

EVEN LEG

$$|N+2\rangle_{1..N+2} = |N\rangle_N |2'\rangle_{N+1,N+2} + |N+1\rangle_{1..N+1} |1\rangle_{N+2}$$

$$|N+2\rangle^p_{1..N+2} = |N+2\rangle_{1..N+2} + |N\rangle_{2..N+1} |2'\rangle_{N+2,1}$$

ODD LEG

$$|N+2\rangle^p_{1..N+2} = |N\rangle_{1..N} |2\rangle_{N+1,N+2} + |N\rangle_{2..N+1} |2\rangle_{N+2,1}$$

CALCULATING ENTANGLEMENT

USING DENSITY MATRIX RECURSION METHOD

RECALL

EVEN LEG

$$|N+2\rangle_{1..N+2} = |N\rangle_N |2'\rangle_{N+1,N+2} + |N+1\rangle_{1..N+1} |1\rangle_{N+2}$$

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ODD LEG

$$|N+2\rangle^p_{1..N+2} = |N\rangle_{1..N} |2\rangle_{N+1,N+2} + |N\rangle_{2..N+1} |2\rangle_{N+2,1}$$

DENSITY MATRIX

EVEN LEG

$$\begin{aligned}\rho^p_{N+2} &= |N+2\rangle\langle N+2|_p \\ &= |N+2\rangle\langle N+2| + |N+2\rangle\langle N|_{2,N+1}\langle 2'|_{N+2,1} \\ &\quad + |N\rangle_{2,N+1}|2'\rangle_{1,N+2}\langle N+2| \\ &\quad + |N\rangle\langle N|_{2,N+1}|2'\rangle\langle 2'|_{N+2,1}\end{aligned}$$

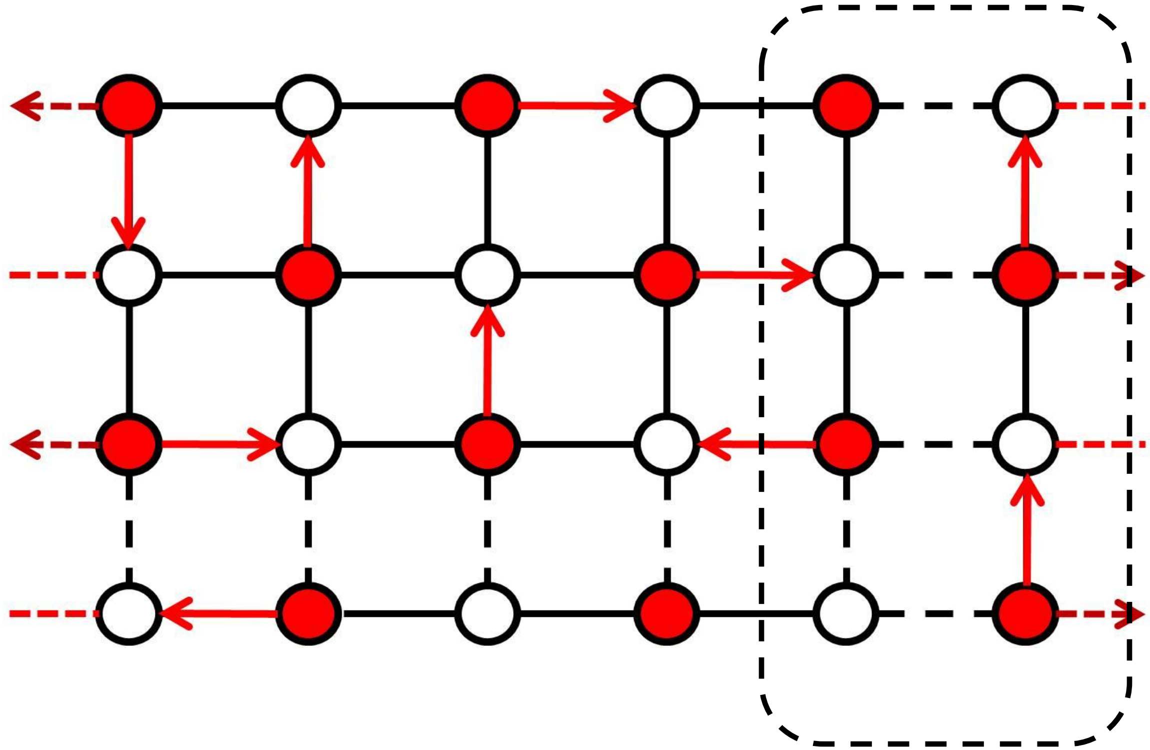
ODD LEG

$$\rho^p_{N+2} = |N+2\rangle\langle N+2|_p$$

CALCULATING ENTANGLEMENT

USING DENSITY MATRIX RECURSION METHOD

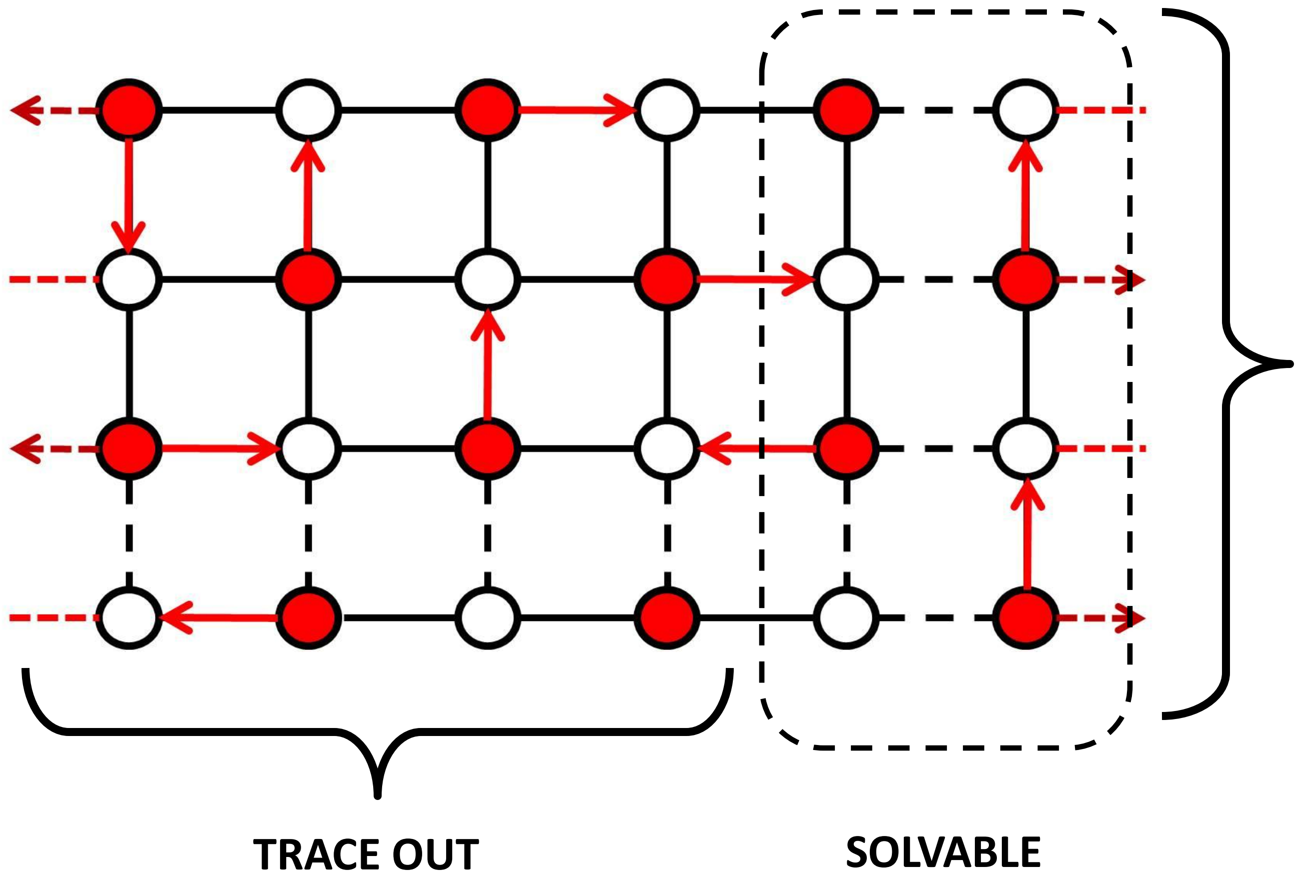
TWO RUNG REDUCED DENSITY MATRIX



CALCULATING ENTANGLEMENT

USING DENSITY MATRIX RECURSION METHOD

TWO RUNG REDUCED DENSITY MATRIX



CALCULATING ENTANGLEMENT

USING DENSITY MATRIX RECURSION METHOD

TWO RUNG REDUCED DENSITY MATRIX

EVEN LEG

$$\rho^{(2)}_{N+1,N+2} = tr_{1..N}(|N+2\rangle\langle N+2|) = tr_{1..N}\big[|N\rangle\langle N||2\rangle\langle 2|_{N+1,N+2} + \big|N-1\rangle\langle N-1||2'\rangle\langle 2'|_{N,N+1}|1\rangle\langle 1|_{N+2} + (|N\rangle|2\rangle_{N+1,N+2}\langle N-1|\langle 2'|_{N,N+1}\langle 1|_{N+2} + h.c.)\big]$$

$$= Z_N|2\rangle\langle 2|_{N+1,N+2} + Z_{N-1}\rho'_{N+1}\otimes|1\rangle\langle 1|_{N+2} + (|2\rangle_{N+1,N+2}\langle \mathcal{E}_N|_{N+1,}\langle 1|_{N+2} + h.c.)$$

$$\rho^{p(2)}_{N+1,N+2} = \rho^{(2)}_{N+1,N+2} + tr_{1..N}\big[|N\rangle\langle N|_{2,N+1}|2'\rangle\langle 2'|_{1,N+2} + (|N\rangle_{2,N+1}|2\rangle_{1,N+2}\langle N+2| + h.c.)\big]$$

$$= \rho^{(2)}_{N+1,N+2} + \beta^{(2)}_{N+1,N+2} + (\gamma^{(2)}_{N+1,N+2} + h.c.)$$

CALCULATING ENTANGLEMENT

USING DENSITY MATRIX RECURSION METHOD

TWO RUNG REDUCED DENSITY MATRIX

$1|_{N+2}$

$1|_{N+2}$

EVEN LEG

$\rangle\langle 1|_{N+2}$

QUANTITIES CAN BE CALCULATED USING ITERATIONS

$= \rho_{N+1,N+2} + \rho_{N+1,N+2} + (\gamma_{N+1,N+2} + h.c.)$

CALCULATING ENTANGLEMENT

USING DENSITY MATRIX RECURSION METHOD

$$\mathcal{Z}_{\mathcal{N}} = \langle \mathcal{N} | \mathcal{N} \rangle = \mathcal{A}\mathcal{Z}_{\mathcal{N}-1} + \mathcal{B}\mathcal{Z}_{\mathcal{N}-2} + 2\mathcal{C}\mathcal{Y}^{1}_{\mathcal{N}-2} + 2\mathcal{D}\mathcal{Y}^{2}_{\mathcal{N}-1}$$

$$\bar{\mathcal{A}} = \langle 1 | \bar{1} \rangle \text{ and } \mathcal{B} = \langle \bar{2} | \bar{2} \rangle$$

$$\langle 1 | \bar{2} \rangle = \mathcal{C} | 1 \rangle + \mathcal{D} | \bar{1} \rangle \text{ and } \langle \bar{1} | \bar{2} \rangle = \bar{\mathcal{C}} | 1 \rangle + \bar{\mathcal{D}} | \bar{1} \rangle$$

$$\langle \xi_{\mathcal{N}-1} |_{n+1} = \langle \bar{2} |_{n,n+1} \langle \mathcal{N} - 2 |_{2..n-1} \mathcal{N} - 1 \rangle_{2..n}.$$

And, $\beta^{2}_{2(n+1,n+2)} = |2\rangle_{n+1,n+2}\langle 1|_{n+1}\langle \xi_{\mathcal{N}}|_{n+2} + |2\rangle_{n+1,n+2}\langle \phi_{\mathcal{N}}|_{n+1,n+2} + \bar{\rho}_{n+1} \otimes |1\rangle_{n+2}\langle \xi_{\mathcal{N}-1}|_{n+2} + \frac{1}{\mathcal{A}}|\xi_1\rangle_{n+1}|1\rangle_{n+2}\langle 1|_{n+1}\langle \eta_{\mathcal{N}-1}|_{n+2}$, where $\langle \phi_{\mathcal{N}}|_{n+1,n+2} = \langle \bar{2}|_{1,n+2}\langle \bar{2}|_{n,n+1}\langle \mathcal{N} - 2|_{2,n-1}\mathcal{N}\rangle$, $\langle \eta_{\mathcal{N}-1}|_{n+2} = \langle \bar{2}|_{1,n+2}\langle \mathcal{N}-1|_{2,n}\mathcal{N}-1\rangle_{1,n-1}|1\rangle_{n}$, and $\mathcal{A} = \langle 1|1\rangle = \langle \bar{1}|\bar{1}\rangle$.

H. S. Dhar, A. Sen (De) and U. Sen, *Density Matrix Recursion Method: Genuine Multisite Entanglement Distinguishes Odd from Even Quantum Heisenberg Ladders*, arXiv:1110.3646v1 [quant-ph] (2011).

CALCULATING ENTANGLEMENT

USING DENSITY MATRIX RECURSION METHOD

$$Z_N = \langle N|N\rangle = AZ_{\ } + BZ_{\ } + 2CY^1_{\ } + 2DY^2_{N-1}$$

An ... +

$|2\rangle$ ... +

$\frac{1}{A}|$ ... =

$\langle\bar{2}|$ ... =

$\langle\bar{2}|_{1,n}$ ... $\langle 1|\bar{1}\rangle.$

ITERATIONS COMPLICATED BUT NOT DIFFICULT TO COMPUTE

CALCULATING ENTANGLEMENT

USING DENSITY MATRIX RECURSION METHOD

ITERATIONS COMPLICATED BUT NOT DIFFICULT TO COMPUTE

AS LONG AS THE BASIC UNITS CAN BE CALCULATED.

CALCULATING ENTANGLEMENT

USING DENSITY MATRIX RECURSION METHOD

$$\mathcal{A},\ \bar{\mathcal{A}},\ \mathcal{B},\ \mathcal{C},\ \bar{\mathcal{C}},\ \mathcal{D},\ \bar{\mathcal{D}},\ \mathcal{Z}_1,\ \mathcal{Y}_1^1,\ \text{and}\ \mathcal{Y}_1^2$$

Specifically, for $M = 2$, the relevant initial parameters are $\mathcal{Z}_0 = 1$, $\mathcal{Z}_1 = 2$, $\mathcal{A} = 2$, $\bar{\mathcal{A}} = 2$, $\mathcal{C} = 1$, $\mathcal{D} = 0$, $\bar{\mathcal{C}} = 0$, $\bar{\mathcal{D}} = 0$, $\mathcal{Y}_1^1 = 2$, and $\mathcal{Y}_1^2 = 0$. For $M = 4$, the initial parameters are $\mathcal{Z}_0 = 1$, $\mathcal{Z}_1 = 4$, $\mathcal{A} = 4$, $\bar{\mathcal{A}} = 2$, $\mathcal{C} = 5$, $\mathcal{D} = 1$, $\bar{\mathcal{C}} = 2$, $\bar{\mathcal{D}} = 3$, $\mathcal{Y}_1^1 = 4$, and $\mathcal{Y}_1^2 = 2$. A similar

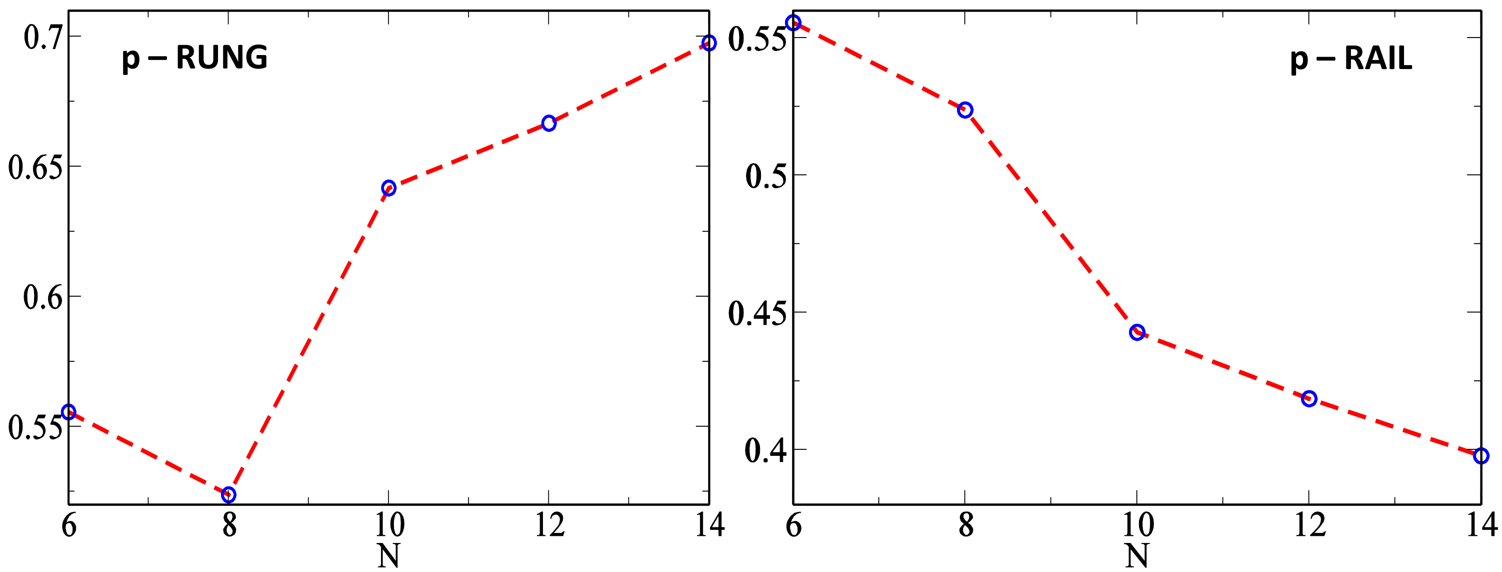
CALCULATING ENTANGLEMENT
USING DENSITY MATRIX RECURSION METHOD

ADVANTAGES:

- CAN BE USED TO CALCULATE LONG MULTI- LEG LADDERS
- CAN BE EXTENDED TO MULTI-LEG LADDERS AS LONG AS BASIC UNIT IS TRACTABLE
- CONVENIENTLY MEASURES MULTISITE REDUCED DENSITY MATRIX.

ENTANGLEMENT PROPERTIES OF TWO-LEG RVB LADDERS

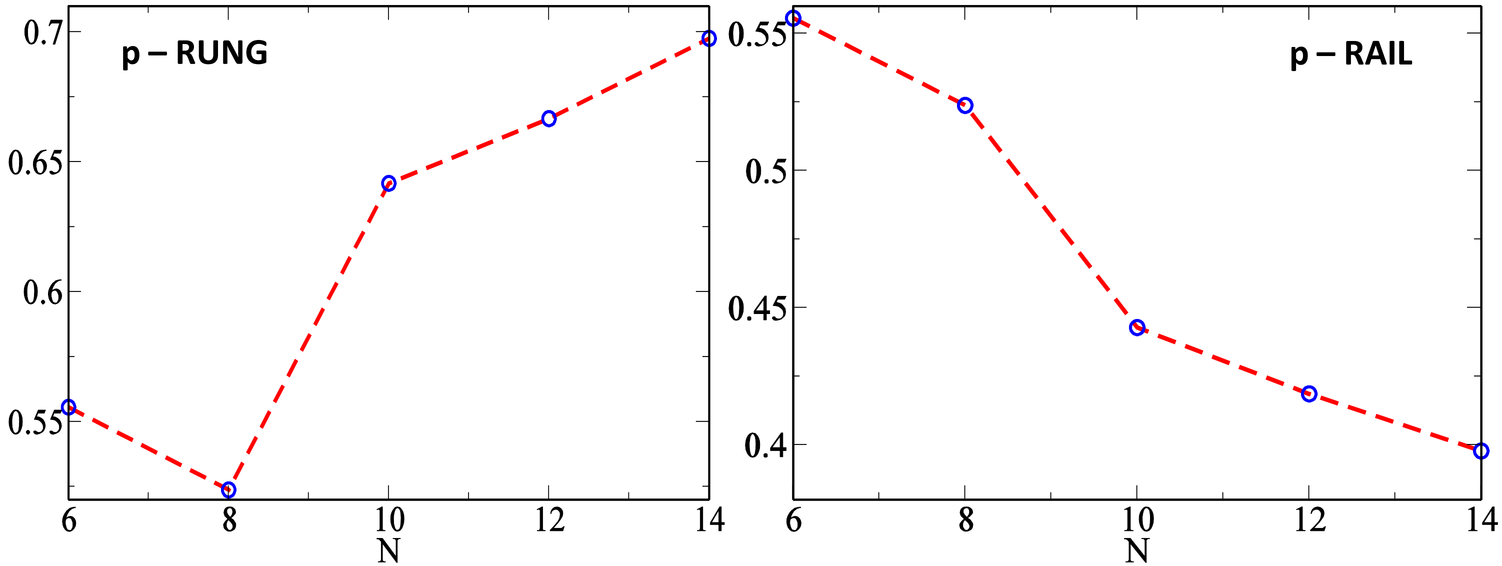
BIPARTITE ENTANGLEMENT



A. Chandran, D. Kaszlikowski, A.Sen De, U. Sen, and V. Vedral, *Phys. Rev. Lett.* **99,** 170502 (2007).
H. S. Dhar and A. Sen De, *J. Phys. A: Math. Theor*. 44 465302 (2011).

ENTANGLEMENT PROPERTIES OF TWO-LEG RVB LADDERS

BIPARTITE ENTANGLEMENT



Results contrast the analytical results obtained for an isotropic 2D or 3D RVB bipartite lattice.

A. Chandran, D. Kaszlikowski, A.Sen De, U. Sen, and V. Vedral, *Phys. Rev. Lett.* **99,** 170502 (2007).
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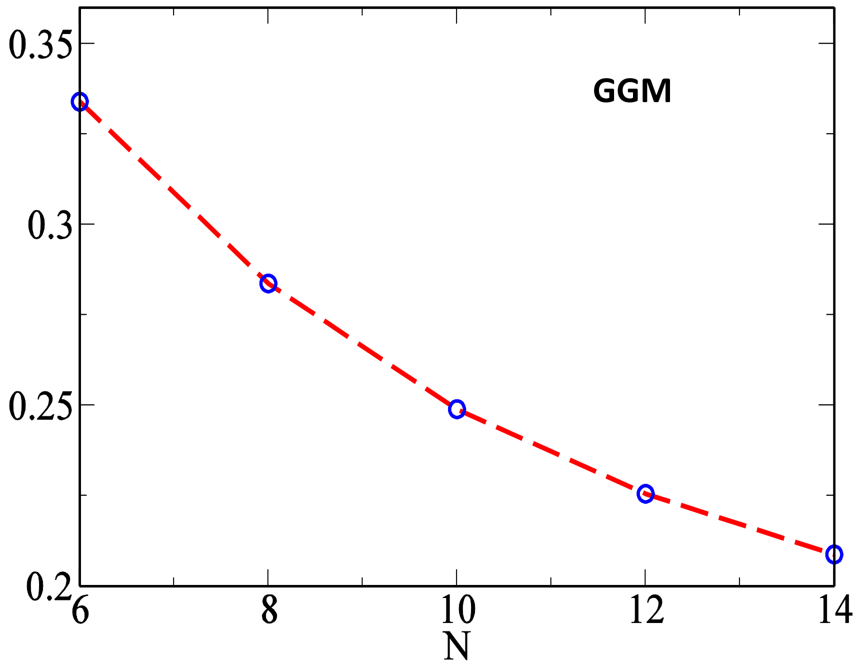
ENTANGLEMENT PROPERTIES OF TWO-LEG RVB LADDERS

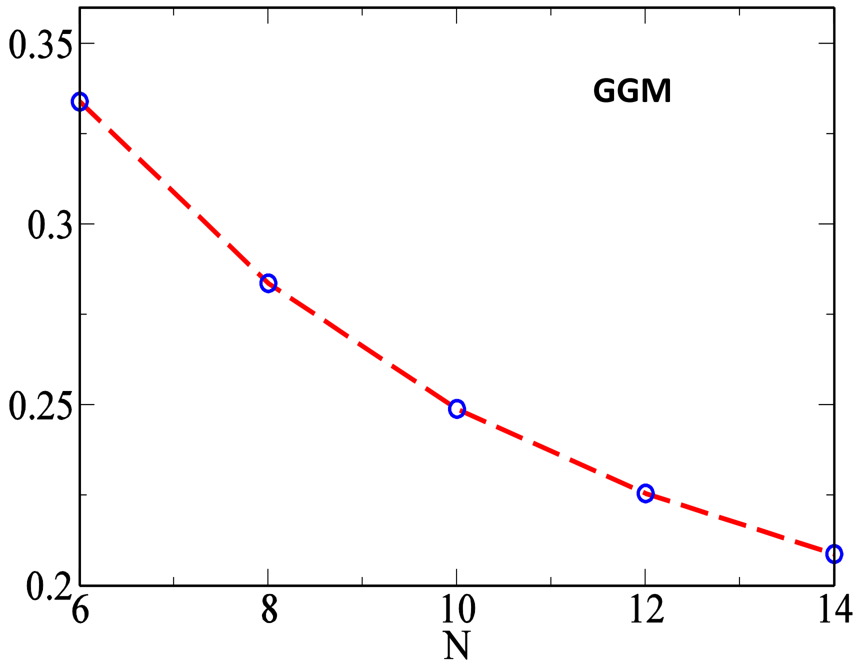
MULTIPARTITE ENTANGLEMENT



A. Chandran, D. Kaszlikowski, A.Sen De, U. Sen, and V. Vedral, *Phys. Rev. Lett.* **99,** 170502 (2007).
H. S. Dhar and A. Sen De, *J. Phys. A: Math. Theor*. 44 465302 (2011).

ENTANGLEMENT PROPERTIES OF TWO-LEG RVB LADDERS

MULTIPARTITE ENTANGLEMENT

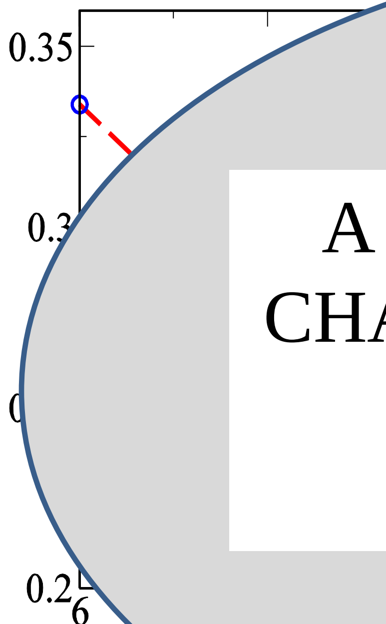


RESULTS ARE AGAIN IN CONTRAST TO THE ISOTROPIC RVB STATE

A. Chandran, D. Kaszlikowski, A.Sen De, U. Sen, and V. Vedral, *Phys. Rev. Lett.* **99,** 170502 (2007).
H. S. Dhar and A. Sen De, *J. Phys. A: Math. Theor*. 44 465302 (2011).

ENTANGLEMENT PROPERTIES OF TWO-LEG RVB LADDERS

MULTIPARTITE ENTANGLEMENT

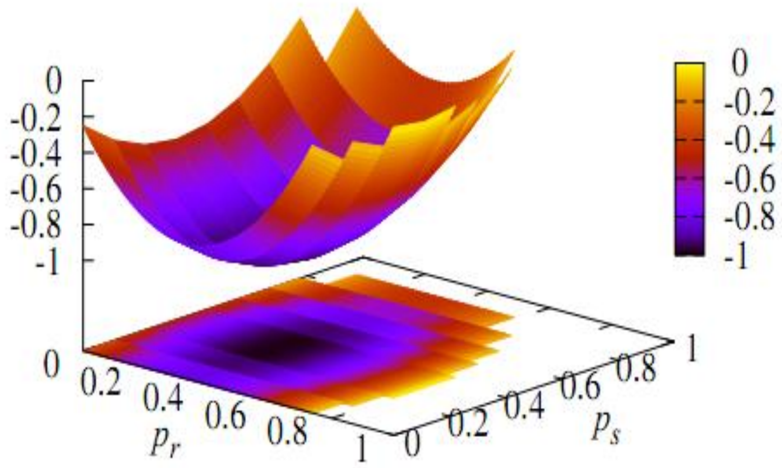


A CHANGE IN GEOMETRY CHANGES THE QUALITATIVE BEHAVIOR OF ENTANGLEMENT

A. Chandran, D. Kaszlikowski, A.Sen De, U. Sen, and V. Vedral, *Phys. Rev. Lett.* **99,** 170502 (2007).
H. S. Dhar and A. Sen De, *J. Phys. A*: *Math. Theor*. 44 465302 (2011).

ENTANGLEMENT PROPERTIES OF TWO-LEG RVB LADDERS

ANALYTICAL BOUNDS

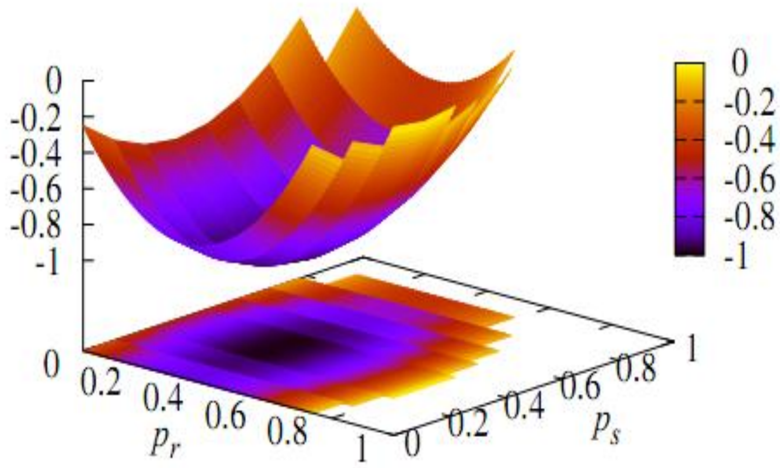


Monogamy bound

H. S. Dhar and A. Sen De, *J. Phys. A: Math. Theor.* 44 465302 (2011).

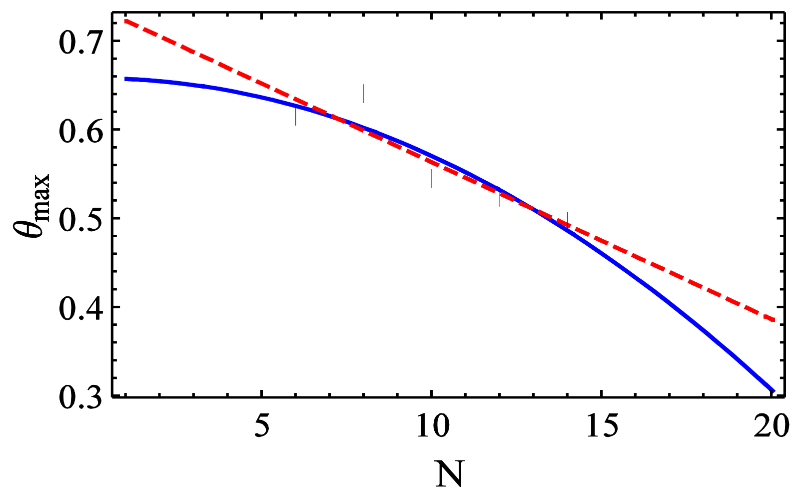
ENTANGLEMENT PROPERTIES OF TWO-LEG RVB LADDERS

ANALYTICAL BOUNDS



Monogamy bound

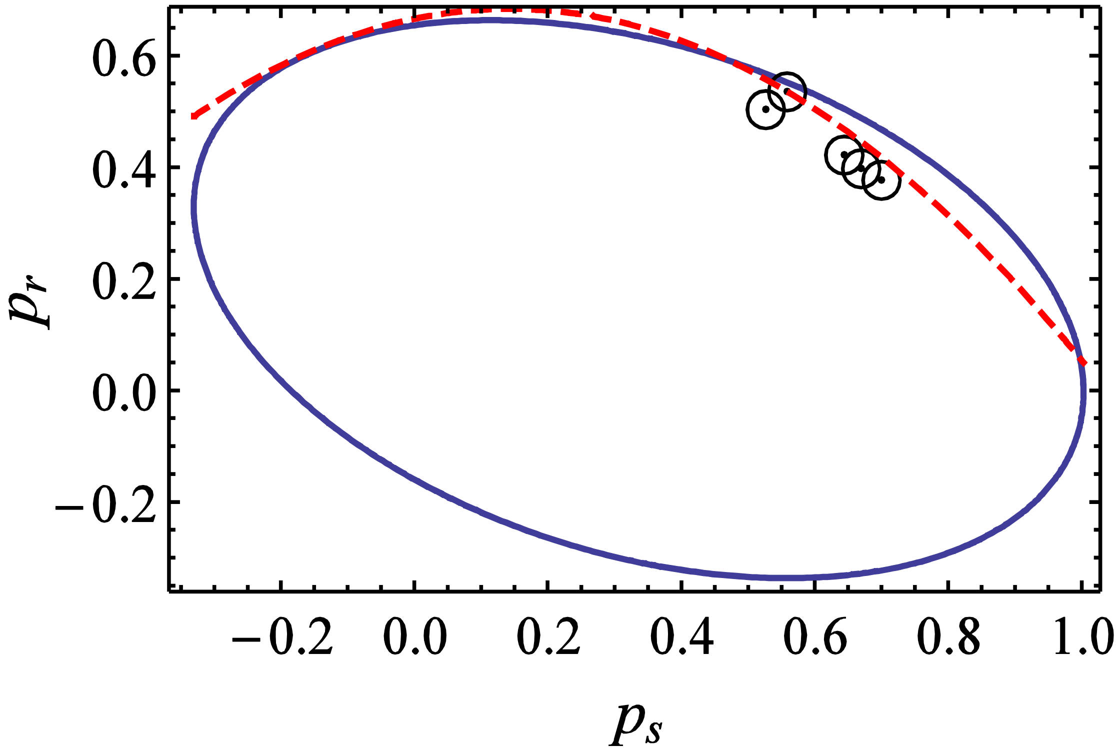
Asymmetric quantum cloning parameter



H. S. Dhar and A. Sen De, *J. Phys. A: Math. Theor*. 44 465302 (2011).

ENTANGLEMENT PROPERTIES OF TWO-LEG RVB LADDERS

ANALYTICAL BOUNDS

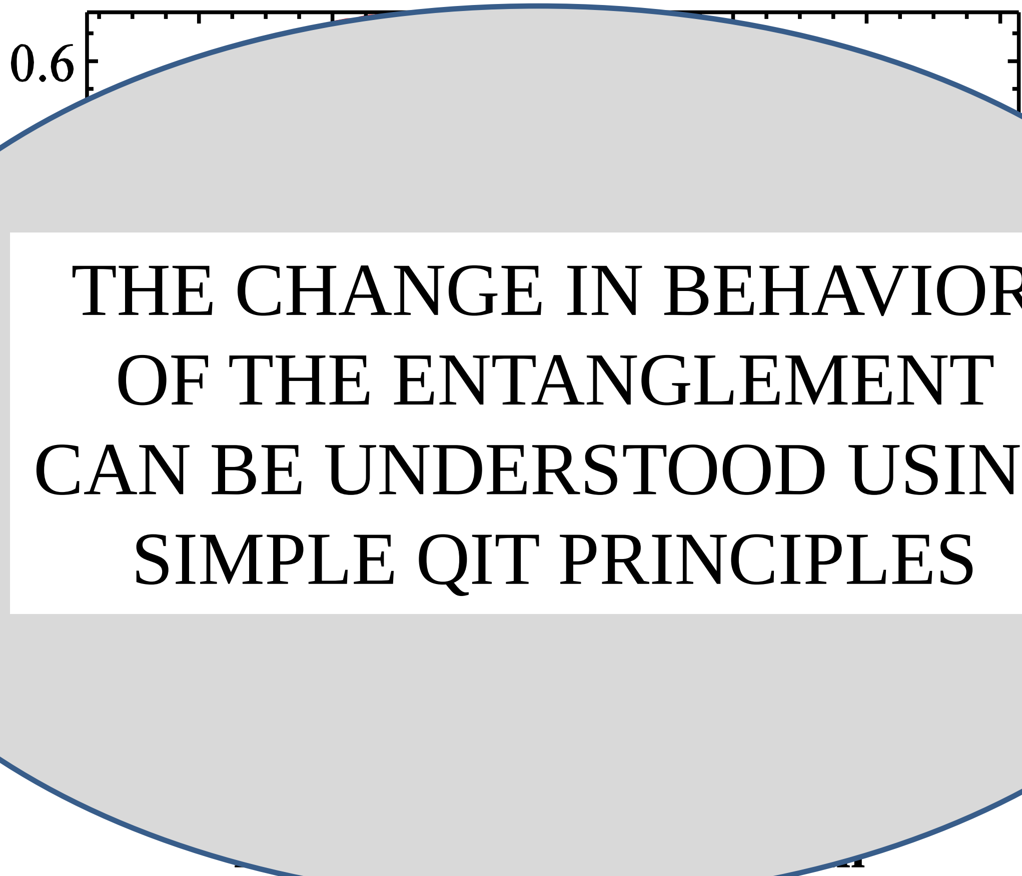


Bound on p-rung and p-rail

H. S. Dhar and A. Sen De, *J. Phys. A: Math. Theor*. 44 465302 (2011).

ENTANGLEMENT PROPERTIES OF TWO-LEG RVB LADDERS

ANALYTICAL BOUNDS



THE CHANGE IN BEHAVIOR OF THE ENTANGLEMENT CAN BE UNDERSTOOD USING SIMPLE QIT PRINCIPLES

H. S. Dhar and A. Sen De, *J. Phys. A: Math. Theor.* 44 465302 (2011).

ENTANGLEMENT PROPERTIES OF MULTI-LEG RVB LADDERS

USING DENSITY MATRIX RECURSION METHOD

DMRM

An analytical iterative technique to obtain the reduced density matrix of an arbitrary number of sites of a quantum spin-1/2 Heisenberg Ladder with an arbitrary number of legs and with both open and periodic boundary conditions.

$$\begin{aligned}|\mathcal{N}+2\rangle &= |\mathcal{N}+1\rangle|1\rangle_{n+2} + |\mathcal{N}\rangle|\bar{2}\rangle_{n+1,n+2} \\ &= |\mathcal{N}\rangle|2\rangle_{n+1,n+2} + |\mathcal{N}-1\rangle|\bar{2}\rangle_{n,n+1}|1\rangle_{n+2}\end{aligned}$$

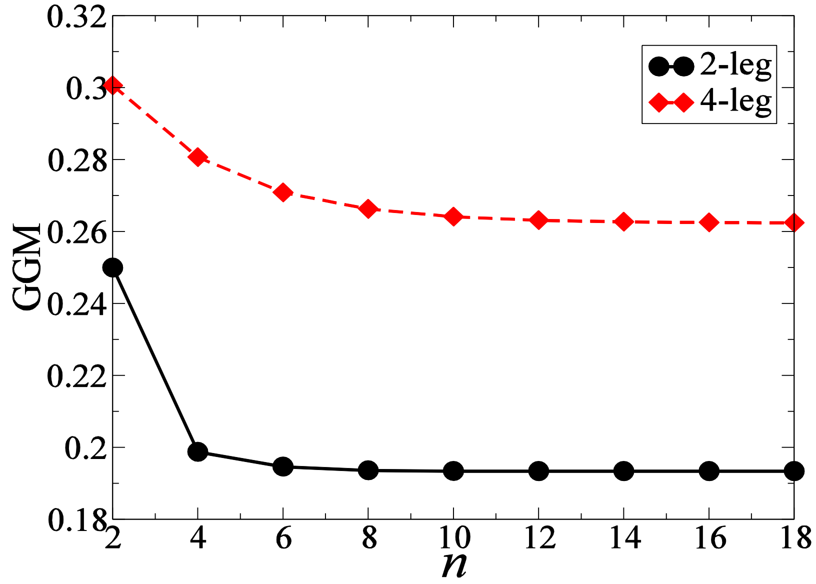
$$\rho^{(2)}_{P(n+1,n+2)} = \mathrm{tr}_{1..n}(|\mathcal{N}+2\rangle\langle\mathcal{N}+2|_P)$$

$$\begin{aligned}\rho^{(2)}_{P(n+1,n+2)} &= \rho^{(2)}_{n+1,n+2} + \mathrm{tr}_{1..n}[|\mathcal{N}\rangle\langle\mathcal{N}|_{(2,n+1)}|\bar{2}\rangle\langle\bar{2}|_{(1,n+2)} \\ &+ (|\mathcal{N}\rangle_{2,n+1}|\bar{2}\rangle_{1,n+2}\langle\mathcal{N}+2| + h.c.)]\end{aligned}$$

H. S. Dhar, A. Sen (De) and U. Sen, *Density Matrix Recursion Method: Genuine Multisite Entanglement Distinguishes Odd from Even Quantum Heisenberg Ladders*, arXiv:1110.3646v1 [quant-ph] (2011).

ENTANGLEMENT PROPERTIES OF MULTI-LEG RVB LADDERS

USING DENSITY MATRIX RECURSION METHOD



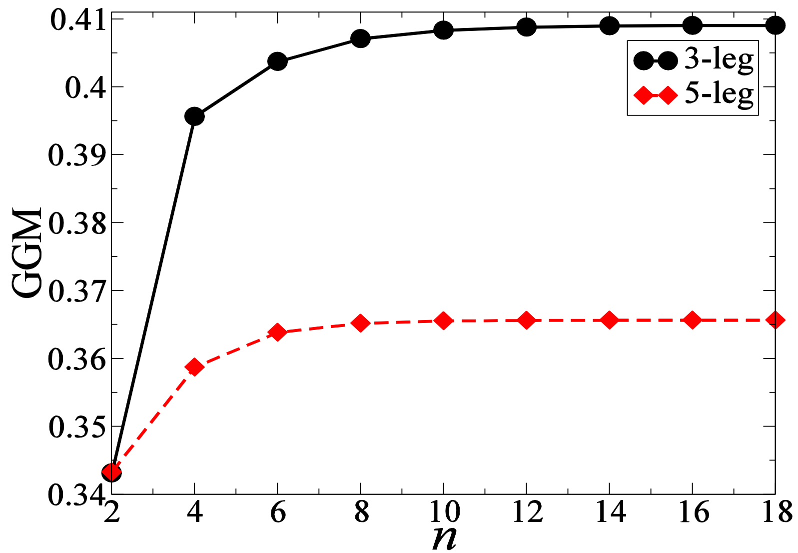
EVEN LEG LADDER

MULTIPARTITE ENTANGLEMENT DECREASES

H. S. Dhar, A. Sen (De) and U. Sen, *Density Matrix Recursion Method: Genuine Multisite Entanglement Distinguishes Odd from Even Quantum Heisenberg Ladders*, arXiv:1110.3646v1 [quant-ph] (2011).

ENTANGLEMENT PROPERTIES OF MULTI-LEG RVB LADDERS

USING DENSITY MATRIX RECURSION METHOD



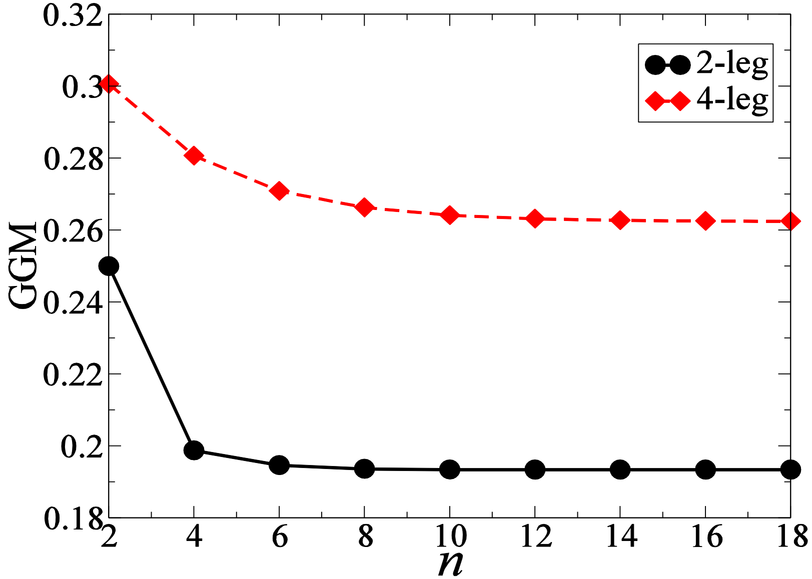
ODD LEG LADDER

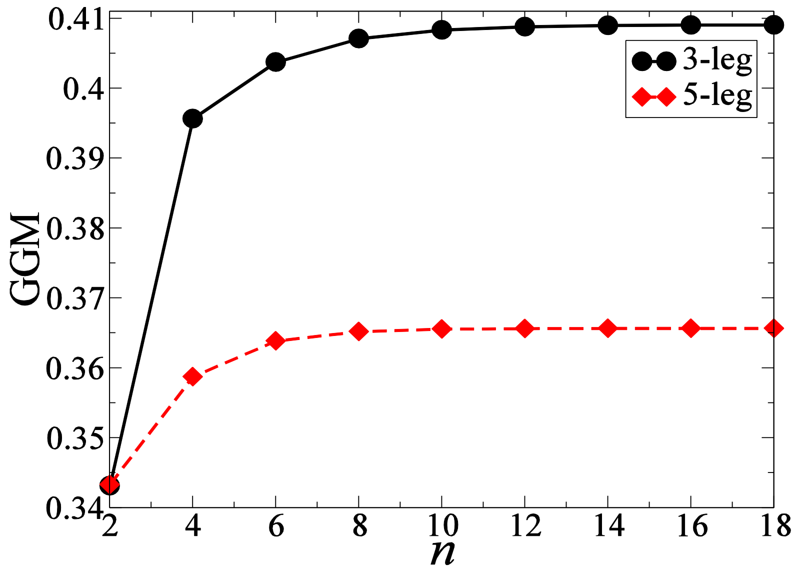
MULTIPARTITE ENTANGLEMENT INCREASES

H. S. Dhar, A. Sen (De) and U. Sen, *Density Matrix Recursion Method: Genuine Multisite Entanglement Distinguishes Odd from Even Quantum Heisenberg Ladders*, arXiv:1110.3646v1 [quant-ph] (2011).

ENTANGLEMENT PROPERTIES OF MULTI-LEG RVB LADDERS

USING DENSITY MATRIX RECURSION METHOD





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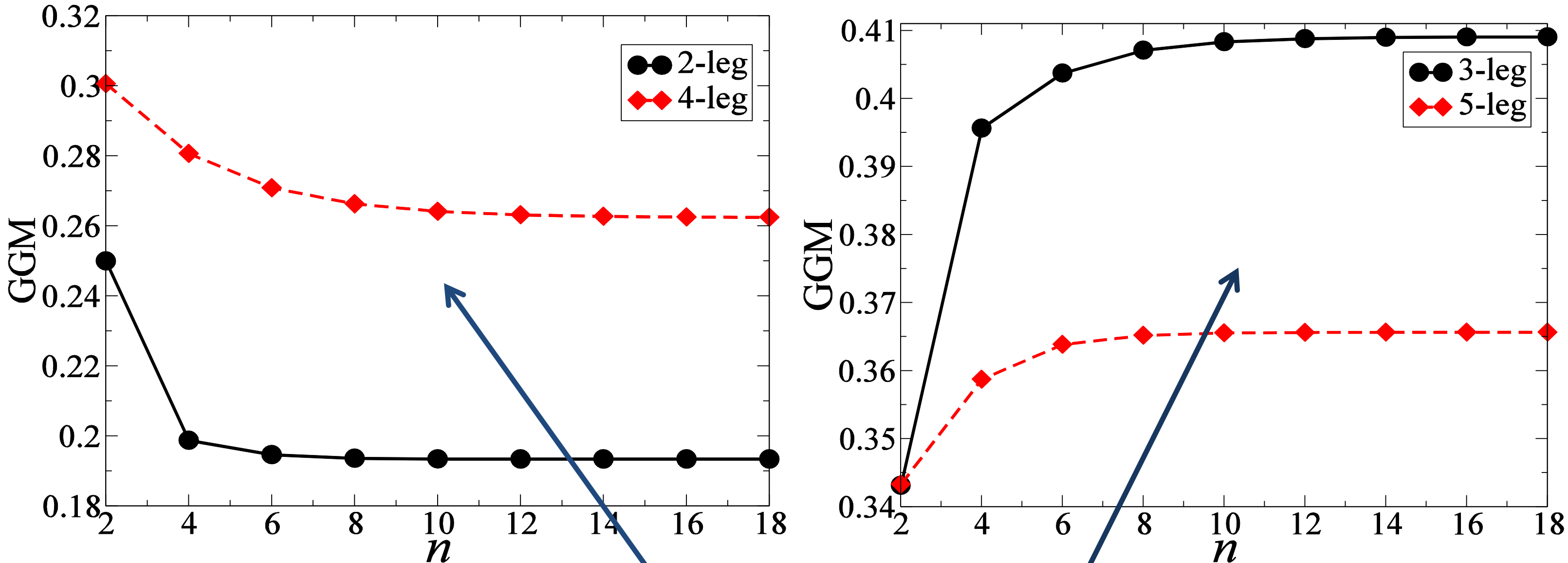
ENTANGLEMENT PROPERTIES OF MULTI-LEG RVB LADDERS

USING DENSITY MATRIX RECURSION METHOD

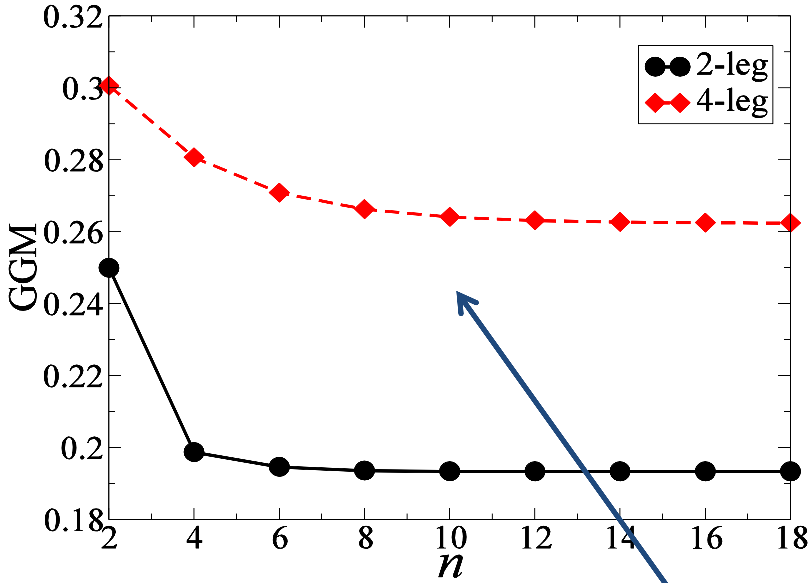


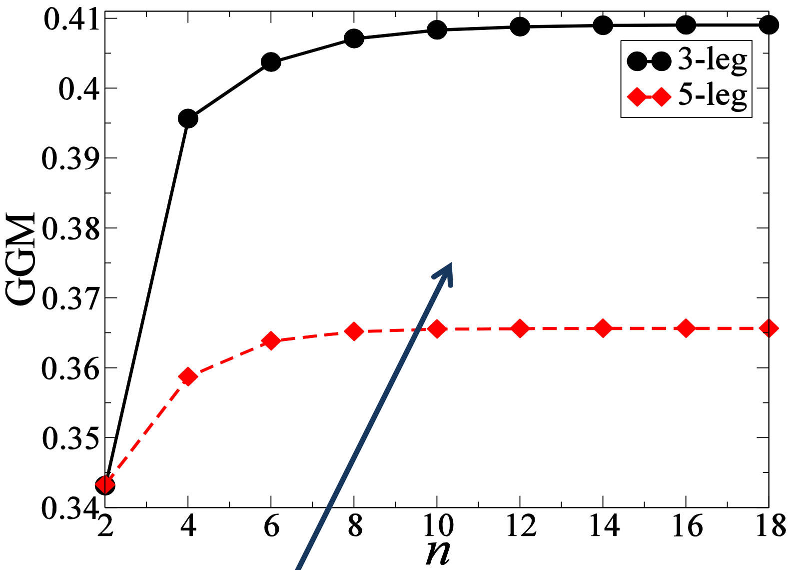
MULTIPARTITE ENTANGLEMENT IS ABLE TO DISTINGUISH EVEN FROM ODD HEISENBERG LADDERS (RVB LADDERS)

H. S. Dhar, A. Sen (De) and U. Sen, *Density Matrix Recursion Method: Genuine Multisite Entanglement Distinguishes Odd from Even Quantum Heisenberg Ladders*, arXiv:1110.3646v1 [quant-ph] (2011).

ENTANGLEMENT PROPERTIES OF MULTI-LEG RVB LADDERS

USING DENSITY MATRIX RECURSION METHOD



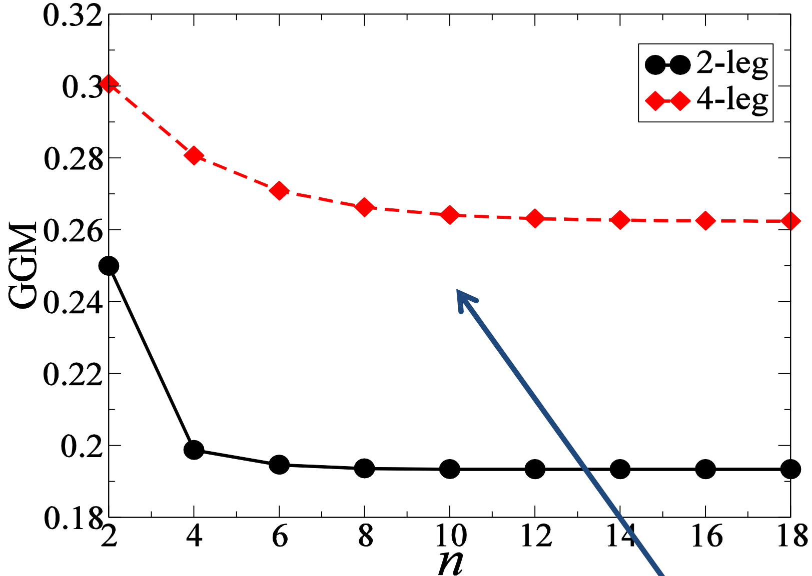


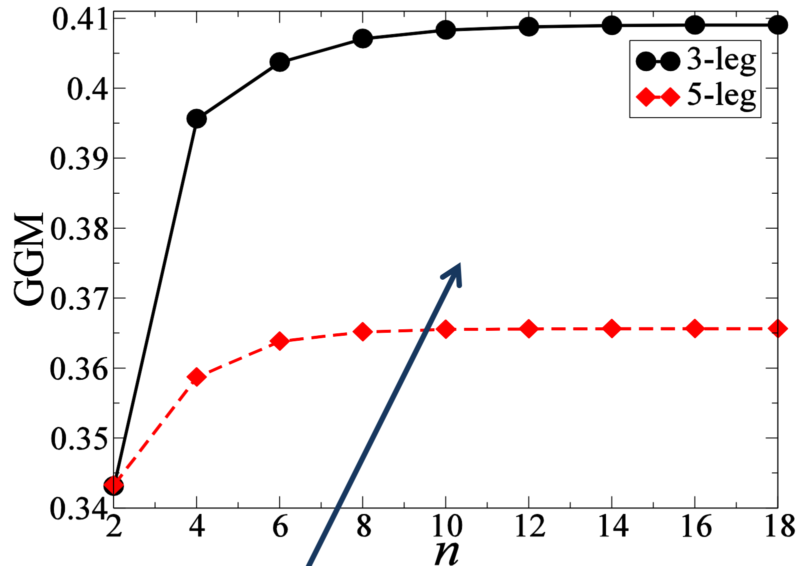
SYSTEM SIZE AS BIG AS **72 SPINS** FOR EVEN AND **90 SPINS** FOR ODD CAN BE EXACTLY CALCULATED USING **DMRM.**

H. S. Dhar, A. Sen (De) and U. Sen, *Density Matrix Recursion Method: Genuine Multisite Entanglement Distinguishes Odd from Even Quantum Heisenberg Ladders*, arXiv:1110.3646v1 [quant-ph] (2011).

ENTANGLEMENT PROPERTIES OF MULTI-LEG RVB LADDERS

USING DENSITY MATRIX RECURSION METHOD



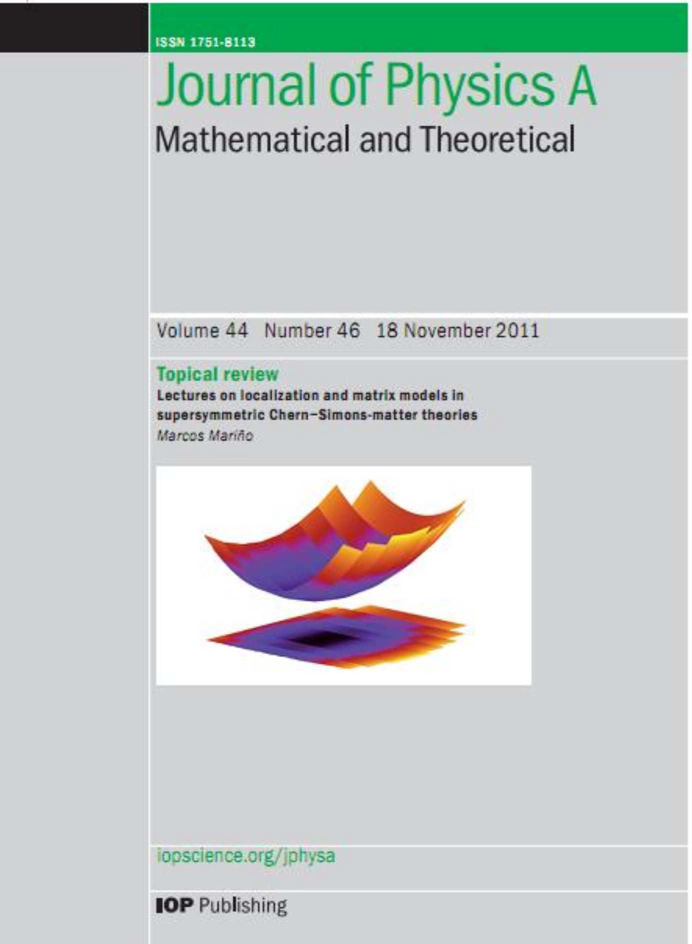


MULTISITE REDUCED DENSITY MATRIX CAN BE CONVENIENTLY CALCULATED

H. S. Dhar, A. Sen (De) and U. Sen, *Density Matrix Recursion Method: Genuine Multisite Entanglement Distinguishes Odd from Even Quantum Heisenberg Ladders,* arXiv:1110.3646v1 [quant-ph] (2011).

IN CONCLUSION

- RVB LADDERS STATES HAVE INTERESTING PROPERTIES
- THE BIPARTITE AND MULTIPARTITE ENTANGLEMENT IS DEPENDENT ON THE GEOMETRY OF THE LADDER
- EXACT SOLUTIONS OF RVB STATES ARE DIFFICULT
- USING A RECURSION TECHNIQUE LIKE DMRM ONE CAN REDUCE THE CALCULATION OF REDUCED DENSITY MATRICES TO ANALYTICAL FORMS
- CAN BE USED TO CALCULATE LARGE SYSTEMS FOR MULTISITE CORRELATIONS



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THANK YOU.